August 17 MATH 1113 sec. 51 Fall 2018

Section 1.2: Relations & Functions

Throughout this course and calculus, we are primarily interested in functions whose domain and range are subsets of the real numbers.

The correspondence (mapping) rule is often expressed by equations providing an algebraic formula telling us how to determine the range element to which a domain element is mapped.

A few preliminary remarks on Function Notation

- We will use a variable character to represent domain elements—usually x (but not always).
- We will use a variable character to represent corresponding range elements—usually y (again, we're not married to y).
- We assign a character name to our functions as well using specific notation—often we use f, sometime g, h or something else.
- The domain and range are not always stated explicitly, but we can often infer them.
- **Reading, writing, and using function notation properly is one of the most critical learning outcomes in this course.**

Function Notation: An example

Consider the equation $y = 3x - 4$. We know that this equation defines a line in the plane. That is, it defines a set of points

$$(x, y) = (x, 3x - 4)$$

where x and y are elements of the set of real[1] numbers $\mathbb{R}$.

The equation $y = 3x - 4$ defines a function. Let's call this function f. We can express this in **function notation** as

$$f(x) = 3x - 4.$$

In English, this reads as

f of *x* equals three *x* minus 4.

[1] The symbol $\mathbb{R}$ denotes the set of all real number.

Function Notation: An example

$$\text{Let} \quad f(x) = 3x - 4, \quad \text{and suppose} \quad y = f(x)$$

- In $f(x)$, f is the function and x is its **argument**.
- x represents an element of the domain, $f(x)$ is an element of the range.
- Since $y = f(x)$, x is called the **independent variable** and y is called the **dependent variable**.
- $y = f(x)$ reads "y equals f of x"
- The collection of points $(x, f(x))$, for each x in the domain, is called **the graph of f**.

Example

Consider the function f defined by $f(x) = -x^2 + 2x + 4$. Evaluate each of the following.

(a) $f(-2) = -(-2)^2 + 2(-2) + 4 = -4 - 4 + 4 = -4$

(b) $f(3a) = -(3a)^2 + 2(3a) + 4 = -9a^2 + 6a + 4$

(c) $f(x+h) = -(x+h)^2 + 2(x+h) + 4 = -(x^2 + 2xh + h^2) + 2x + 2h + 4$

$= -x^2 - 2xh - h^2 + 2x + 2h + 4$

Question

Let $f(x) = -x^2 + 2x + 4$. Evaluate $f(3)$ and $f(-b)$.

(a) $f(3) = 1$ and $f(-b) = -3b + 4$

(b) $f(3) = 7$ and $f(-b) = -b^2 + 2b + 4$

(c) $f(3) = 1$ and $f(-b) = b^2 - 2b + 4$

(d) $f(3) = 1$ and $f(-b) = -b^2 - 2b + 4$

$f(3) = -(3)^2 + 2(3) + 4$

$= -9 + 6 + 4 = 1$

$f(-b) = -(-b)^2 + 2(-b) + 4$

$= -b^2 - 2b + 4$

$* \quad -x^2 = -x \cdot x$

$(-x)^2 = (-x) \cdot (-x)$

Example: $f(x) = -x^2 + 2x + 4$

Assume h is a nonzero real number. Simplify the expression

$$\frac{f(x+h) - f(x)}{h}$$

from before $f(x+h) = -x^2 - 2xh - h^2 + 2x + 2h + 4$

$$\frac{f(x+h) - f(x)}{h} = \frac{-x^2 - 2xh - h^2 + 2x + 2h + 4 - (-x^2 + 2x + 4)}{h}$$

$$= \frac{\cancel{-x^2} - 2xh - h^2 + \cancel{2x} + 2h + \cancel{4} + \cancel{x^2} - \cancel{2x} - \cancel{4}}{h}$$

$$= \frac{-2xh - h^2 + 2h}{h}$$

$$= \frac{\cancel{h}(-2x - h + 2)}{\cancel{h}}$$

$$= -2x - h + 2$$

Graph of $f(x) = -x^2 + 2x + 4$

x	$f(x)$	$(x, f(x))$
$-\frac{3}{2}$	$-\frac{5}{4}$	$(-\frac{3}{2}, -\frac{5}{4})$
-1	1	$(-1, 1)$
0	4	$(0, 4)$
1	5	$(1, 5))$
$\frac{3}{2}$	$\frac{19}{4}$	$(\frac{3}{2}, \frac{19}{4})$
3	1	$(3, 1)$

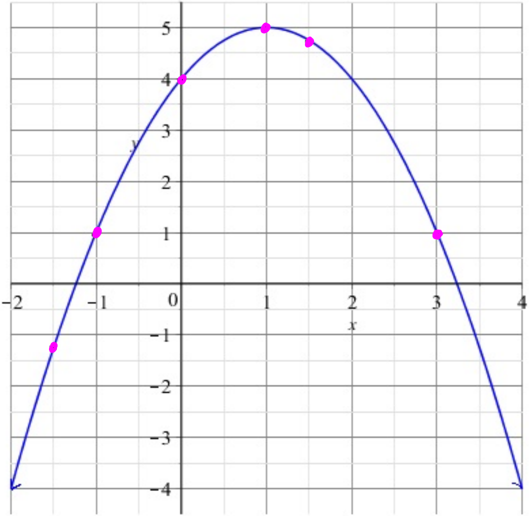
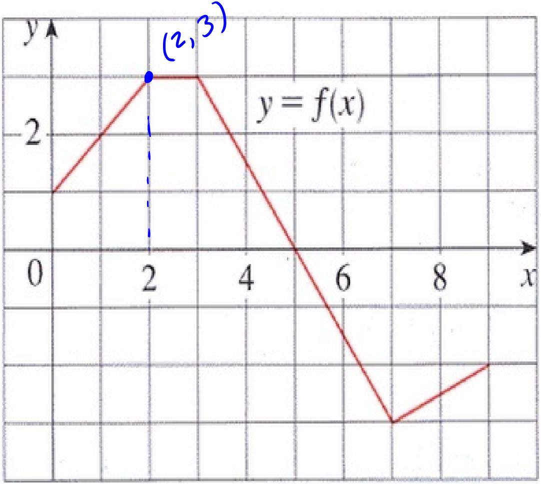


Figure: A table with several sample values and a graph of $y = f(x)$.

Question

From the graph of $y = f(x)$, evaluate $f(2)$



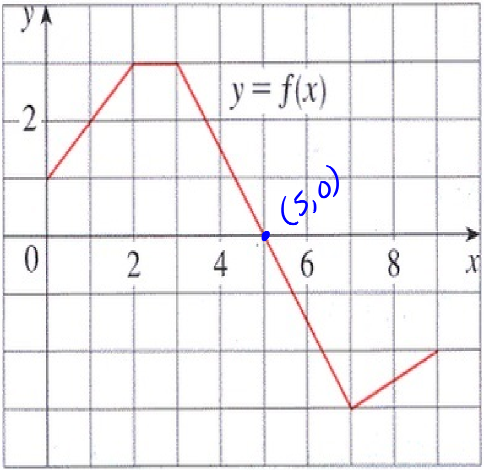
(a) 1

(b) 1 and 3.6

(c) 3

Question

From the graph of $y = f(x)$, if $f(x) = 0$, then $x =$



(a) 1

(b) 5

(c) can't be determined

Vertical Line Test

The graph of a function can be intersected at most one time by any vertical line.

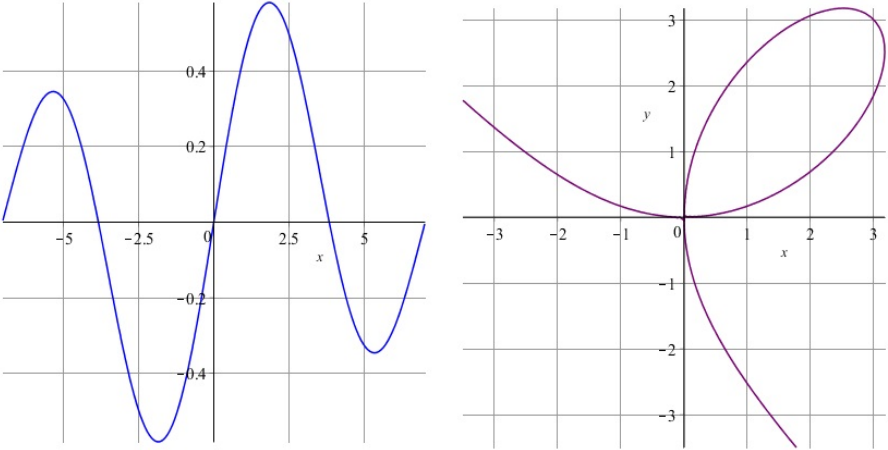
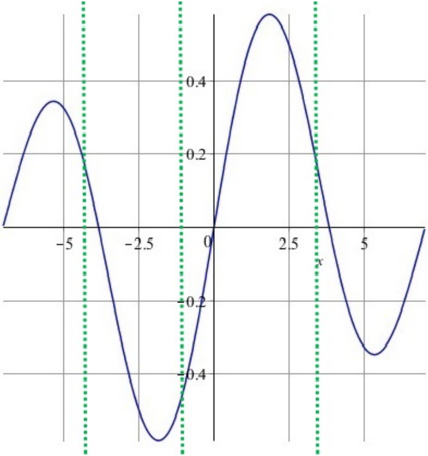


Figure: Plots of two relations. One is a function, the other is not.

Vertical Line Test

The graph of a function can be intersected at most one time by any vertical line.



Note each vertical line intersects the graph at most one time.

Figure: Plot of a function. No vertical line intersects the graph in more than one point.

Vertical Line Test

The graph of a function can be intersected at most one time by any vertical line.

Note that the line $x = 1$ intersects the graph more than one time (3-times)

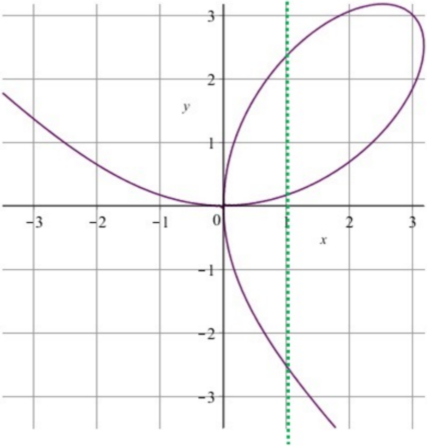


Figure: Plot of a relation that is NOT a function. We can find at least one vertical line that intersects the graph in more than one point.

Domain & Range

Unless stated otherwise, the domain of a function defined by an equation $y = f(x)$ is assumed to be the largest subset of the real numbers for which the value $f(x)$ is defined. In general, we eliminate any real numbers for which $f(x)$ is not defined as a real number. Recall

- division by zero is not defined
- negative numbers do not have any even roots (square root, fourth root, etc.)
- other *function properties* are (or will be) known such as negative numbers having no logarithms

Example

Determine the domain of each function.

(a) $f(x) = -x^2+2x+4$

Are there any real numbers x for which $f(x)$ would not be defined? No!

The domain is all real numbers.

Symbolically the domain is $\mathbb{R}$.

In interval notation, the domain is

$(-\infty, \infty)$.

Example

Determine the domain of each function.

(b) $f(x) = \dfrac{\sqrt{x}}{x-1}$

For the domain, we need $\sqrt{x}$ defined i.e. $x \geq 0$.

Because of the denominator, we need $x - 1 \neq 0$
(no division by zero)

The domain is $x \geq 0$ with $x \neq 1$.

In interval notation

$$[0, 1) \cup (1, \infty)$$

Question

The domain of $f(x) = \dfrac{x^2}{\sqrt[4]{x+3}}$ is

(a) $(-3, \infty)$

(b) $(-2, 0) \cup (0, \infty)$

(c) $[-3, \infty)$

(d) $(-\infty, -3) \cup (-3, \infty)$

We require $x+3 > 0$

$x > -3$

$(-3, \infty)$