August 20 MATH 1113 sec. 51 Fall 2018

Section 1.2: Relations & Functions

Domain & Range Unless stated otherwise, the domain of a function defined by an equation $y = f(x)$ is assumed to be the largest subset of the real numbers for which the value $f(x)$ is defined. In general, we eliminate any real numbers for which $f(x)$ is not defined as a real number. Recall

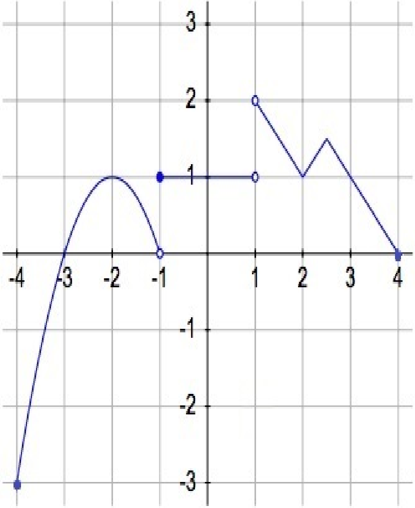
- division by zero is not defined
- negative numbers do not have any even roots (square root, fourth root, etc.)
- other *function properties* are (or will be) known such as negative numbers having no logarithms

Domain & Range

- The range may be difficult to infer from a formula. Sometimes it is possible by recalling known properties—e.g. $|x|$ is always nonnegative.
- The domain and range can often be determined from a graph.
- Recall that the range is the set of all possible $f(x)$—i.e.y—values.

Domain & Range from a Graph

Domain → x-values



The left and right most x's are -4 and 4.

all x in [-4, -1) are included.

all x in [-1, 1) are included.

all x in (1, 4] are included.

The domain is

$$[-4, 1) \cup (1, 4]$$

Figure: Identify the domain from the plot of $y = f(x)$

Domain & Range from a Graph

range → y-values

The lowest and highest y-values are −3 and 2.

all y on [-3, 1] are included. The y-values on [1, 2) are also included. The range is

[-3, 2).

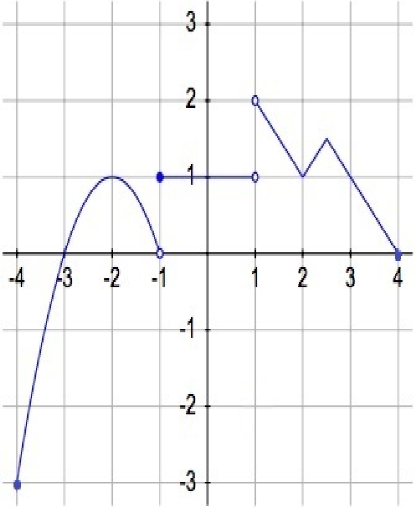
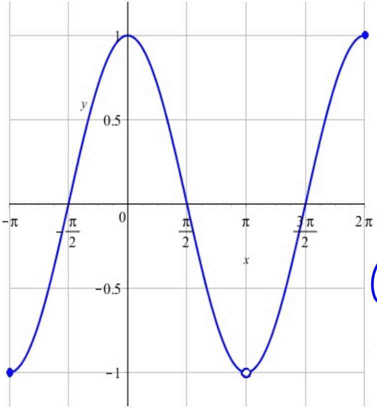


Figure: Identify the range from the plot of $y = f(x)$

Question

Identify the domain and range from the graph of $y = f(x)$.



(a) Domain is $(-\pi, 2\pi)$, Range is $(-1, 1)$

(b) Domain is $[-\pi, 2\pi]$, Range is $[-1, 1]$

(c) Domain is $[-\pi, \pi) \cup (\pi, 2\pi]$, Range is $(-1, 1]$

(d) Domain is $[-\pi, \pi) \cup (\pi, 2\pi]$, Range is $[-1, 1]$

(e) can't be determined without more information

Section 2.2: The Algebra of Functions

Calculus Alert: Being fluent in the operations and the notation will help you be successful in Calculus.

We can create new functions from old by combining them with the operations of addition, subtraction, multiplication, division, and composition.

Addition, Subtraction, Multiplication, and Division of Functions

Let f and g be functions, and suppose that x is in the domain of each. Then define $f+g$, $f-g$, fg and f/g, and use the following notation

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x)g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ provided $g(x) \neq 0$

Domain

For functions f and g, the domain of $f+g$, $f-g$, and fg is the set of all x such that x is in the domain of f and x is in the domain of g.

the intersection of the domains of f and g

The domain of f/g is the set of all all x such that x is in the domain of f, x is in the domain of g, and $g(x) \neq 0$.

the domain of fg less any x such that $g(x) = 0$

Example

Let $f(x) = \sqrt{x+1}$ and $g(x) = 3x^2$. Evaluate

(a) $(f+g)(3)$ $= f(3) + g(3) = \sqrt{3+1} + 3(3^2) = \sqrt{4} + 3(9) = 2+27$
$= 29$

(b) $(f-g)\left(-\frac{3}{4}\right)$ $= f\left(\frac{-3}{4}\right) + g\left(\frac{-3}{4}\right) = \sqrt{\frac{-3}{4}+1} + 3\left(\frac{-3}{4}\right)^2$
$= \frac{1}{2} + \frac{27}{16} = \frac{35}{16}$

(c) $(f+g)(x)$

$= f(x) + g(x) = \sqrt{x+1} + 3x^2$