August 22 MATH 1113 sec. 51 Fall 2018

Section 2.2: The Algebra of Functions

Let f and g be functions, and suppose that x is in the domain of each. Then define f + g, f - g, fg and f/g, and use the following notation

$$(f+g)(x) = f(x) + g(x)$$

►
$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\qquad \qquad \bullet \ \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ provided } g(x) \neq 0$$



Domain

For functions f and g, the domain of f + g, f - g, and fg is the set of all x such that x is in the domain of f and x is in the domain of g.

the intersection of the domains of f and g

The domain of f/g is the set of all all x such that x is in the domain of f, x is in the domain of g, and $g(x) \neq 0$.

the domain of fg less any x such that g(x) = 0

Example

Let $f(x) = \sqrt{x+1}$ and $g(x) = 3x^2$. Evaluate

(a)
$$(f+g)(3) = f(3) + g(3) = \sqrt{3+1} + 3(3^2) = \sqrt{4} + 3(9) = 2+27$$

(b)
$$(f-g)(-\frac{3}{4})$$

$$= \int (-\frac{3}{4}) - g(-\frac{3}{4})$$

$$= \int (\frac{3}{4}) - g(-\frac{3}{4})^2 = \int \frac{1}{4} - 3(\frac{9}{16})$$

$$= \frac{1}{4} - \frac{27}{16} = \frac{8-27}{16} = \frac{-19}{16}$$

Example

Let $f(x) = \sqrt{x+1}$ and $g(x) = 3x^2$. Identify the domains of f+g and f/g.

For x in the domain of f, we require X+170 = X>-1. In interval notation the domain of f is [-1,20). The domain of g is all reals, i.e. (-10,00). The intersection is (1,00). The domain of ftg is [-1, so).

$$f(x) = \sqrt{x+1} , \quad g(x) = 3x^{2}$$
The domain of frg is [-1, \infty].

Note $g(x) = 0 \Rightarrow 3x^{2} = 0 \Rightarrow \infty = 0$.

The domain of $\frac{f}{g}$ is

$$[-1, 0) \cup (0, \infty).$$

Question

Let
$$f(x) = 2x^2$$
 and $g(x) = \frac{2}{x-5}$. Evaluate $(fg)(2)$ and $(\frac{f}{g})(1)$.

(a)
$$(fg)(2) = -3$$
 and $(\frac{f}{g})(1) = -4$

(b)
$$(fg)(2) = -\frac{16}{3}$$
 and $(\frac{f}{g})(1) = -1$

(c)
$$(fg)(2) = -\frac{16}{3}$$
 and $(\frac{f}{g})(1) = -4$

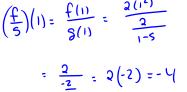
(d)
$$(fg)(2) = \frac{16}{3}$$
 and $(\frac{f}{g})(1) = -1$

(d)
$$(fg)(2) = \frac{16}{2}$$
 and $(f)(1) = -1$

$$(f_6)^{(2)} = f(2)g(2)$$

= $g(2^2) \left(\frac{2}{2-5}\right)$

$$= 3.4 \left(\frac{2}{-3}\right) = \frac{-16}{3}$$



August 21, 2018

6/43

Section 2.3: Compositions

Suppose a spherical balloon is inflated so that the radius after time t seconds is given by the function r(t) = 2t cm. The volume of a sphere of radius r is known to be $V(r) = \frac{4}{3}\pi r^3$. Note that

- r is a function of t, and
- V is a function of r, making
- ▶ *V* a function of *t* (through its dependence on *r*). In fact,

$$V(t) = V(r(t)) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3.$$

This is an example of a **composition** of functions.

Composition: Definition and Notation

Let f and g be functions. Then the **composite** function denoted

$$f \circ g$$
,

also called the **composition** of f and g, is defined by

$$(f\circ g)(x)=f(g(x)).$$

The domain of $f \circ g$ is the set of all x in the domain of g such that g(x) is in the domain of f.

The expression $f \circ g$ is read "f composed with g", and $(f \circ g)(x)$ is read "f of g of x".

Example

Let $f(x) = \sqrt{x-1}$ and $g(x) = \frac{2}{x+1}$. Evaluate each expression if possible.

(a)
$$(f \circ g)(1) : f(g(1)) : f(\frac{2}{1+1}) : f(\frac{2}{2}) : f(1) : \sqrt{1-1} = 0$$

(b)
$$(f \circ g)(0) = f(g(g)) = f(z) = \sqrt{z-1} = \sqrt{1-z}$$

(c)
$$(g \circ f)(0) = g(f(0)) = g(J_{0-1}) = g(J_{-1})$$
 undefined
O is not in the donoise of gof
It's not in the donoise of f either.



Question

Let $f(x) = \sqrt{x-1}$ and $g(x) = \frac{2}{x+1}$. Evaluate $(g \circ f)(1)$ if possible.

(a)
$$(g \circ f)(1) = 0$$

(b)
$$(g \circ f)(1) = 1$$

$$(c) g \circ f)(1) = 2$$

(d) $(g \circ f)(1)$ is undefined

$$=g(0)=\frac{2}{0+1}=2$$