August 22 MATH 1113 sec. 52 Fall 2018

Section 2.2: The Algebra of Functions

Let f and g be functions, and suppose that x is in the domain of each. Then define $f+g$, $f-g$, fg and f/g, and use the following notation

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x)g(x)$
- $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$ provided $g(x) \neq 0$

Domain

For functions f and g, the domain of $f+g$, $f-g$, and fg is the set of all x such that x is in the domain of f and x is in the domain of g.

the intersection of the domains of f and g

The domain of f/g is the set of all all x such that x is in the domain of f, x is in the domain of g, and $g(x) \neq 0$.

the domain of fg less any x such that $g(x) = 0$

Example

Let $f(x) = \sqrt{x+1}$ and $g(x) = 3x^2$. Evaluate

(a) $(f+g)(3) = f(3) + g(3) = \sqrt{3+1} + 3(3^2) = \sqrt{4} + 3(9)$

$= 2 + 27 = 29$

(b) $(f-g)\left(-\frac{3}{4}\right)$

$= f\left(\frac{-3}{4}\right) - g\left(\frac{-3}{4}\right)$

$= \sqrt{\frac{-3}{4}+1} - 3\left(\frac{-3}{4}\right)^2 = \sqrt{\frac{1}{4}} - 3\left(\frac{9}{16}\right) = \frac{1}{2} - \frac{27}{16}$

$= \frac{8-27}{16} = \frac{-19}{16}$

Example

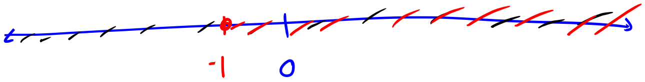
Let $f(x) = \sqrt{x+1}$ and $g(x) = 3x^2$. Identify the domains of $f+g$ and f/g.

For x in the domain of f, we require

$x+1 \geq 0 \implies x \geq -1$. In interval notation

the domain of f is $[-1, \infty)$.

The domain of g is all reals; in interval notation $(-\infty, \infty)$.



The domain of $f+g$ is $[-1,\infty)$.

For the domain of $\frac{f}{g}$, note that the equation $g(x)=0 \implies 3x^2=0$ has one solution $x=0$. So the domain of $\frac{f}{g}$ is the interval $[-1,\infty)$ <u>without</u> $x=0$.

$[-1,0)\cup(0,\infty)$ is the domain of $\frac{f}{g}$.

Question

Let $f(x) = 2x^2$ and $g(x) = \dfrac{2}{x-5}$. Evaluate $(fg)(2)$ and $\left(\dfrac{f}{g}\right)(1)$.

(a) $(fg)(2) = -3$ and $\left(\dfrac{f}{g}\right)(1) = -4$

(b) $(fg)(2) = -\frac{16}{3}$ and $\left(\dfrac{f}{g}\right)(1) = -1$

(c) $(fg)(2) = -\frac{16}{3}$ and $\left(\dfrac{f}{g}\right)(1) = -4$

(d) $(fg)(2) = \frac{16}{3}$ and $\left(\dfrac{f}{g}\right)(1) = -1$

$(fg)(2) = f(2)\,g(2)$

$= 2(2^2)\left(\dfrac{2}{2-5}\right)$

$= 2\cdot 4\cdot \dfrac{2}{-3} = \dfrac{-16}{3}$

$\left(\dfrac{f}{g}\right)(1) = \dfrac{f(1)}{g(1)} = \dfrac{2(1^2)}{\frac{2}{1-5}}$

$= \dfrac{2}{\frac{2}{-4}} = \dfrac{2}{-1/2} = 2(-2) = -4$