August 24 MATH 1113 sec. 51 Fall 2018

Section 2.3: Compositions

Let f and g be functions. Then the **composite** function denoted

$$f \circ g,$$

also called the **composition** of f and g, is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

The expression $f \circ g$ is read "f composed with g", and $(f \circ g)(x)$ is read "f of g of x".

Example $f(x) = \sqrt{x-1}$ and $g(x) = \frac{2}{x+1}$

Find a simplified formula for $F(x) = (f \circ g)(x)$ and determine its domain.

$$F(x) = (f \circ g)(x) = f(g(x)) = f\left(\frac{2}{x+1}\right) = \sqrt{\frac{2}{x+1} - 1}$$

we can simplify a little

$$F(x) = \sqrt{\frac{2}{x+1} - \frac{x+1}{x+1}} = \sqrt{\frac{2-(x+1)}{x+1}} = \sqrt{\frac{2-x-1}{x+1}}$$

$$= \sqrt{\frac{1-x}{x+1}}$$

Note the domain of $f(x)$ is $x \geq 1$ i.e. $[1, \infty)$.

The domain of $g(x)$ is $(-\infty, -1) \cup (-1, \infty)$.

$F(x) = \sqrt{\dfrac{1-x}{x+1}}$. For the domain, we require $x \neq -1$, and $\dfrac{1-x}{x+1} \geq 0$. So $1-x$ and $x+1$ have to have the same sign.

If $1-x \geq 0$ and $x+1 \geq 0$ then

$x \leq 1$ $\quad$ $x \geq -1$, with $x \neq -1$

we have $-1 < x \leq 1$. If $1-x < 0$ and $x+1 < 0$

Then $x > 1$ and $x < -1$ $\leftarrow$ no intersection!

The domain of $f \circ g$ is the interval

$$(-1, 1].$$

Example $f(x) = \sqrt{x-1}$ and $g(x) = \frac{2}{x+1}$

Find a simplified formula for $H(x) = (g \circ f)(x)$, and determine its domain.

$$H(x) = (g \circ f)(x) = g(f(x)) = g(\sqrt{x-1})$$

$$= \frac{2}{\sqrt{x-1}+1}$$

$$H(x) = \frac{2}{\sqrt{x-1}+1},$$ for the domain

we need $x-1 \geq 0 \implies x \geq 1$

And, we require $\sqrt{x-1}+1 \neq 0$

If we try to solve $\sqrt{x-1}+1=0$

we get $\sqrt{x-1}=-1$. Since $\sqrt{x-1}\geq 0$

for all x, the equation has no solutions.

The only condition is $x \geq \underline{1}$, so

the domain of H is the interval

$[1,\infty)$.

Question

The function $p(x) = \dfrac{1}{(x+3)^5}$ could be the composition $f \circ g$ of which pair of functions?

(a) $f(x) = x^5$, and $g(x) = \frac{1}{x+3}$

(b) $f(x) = \frac{1}{x+3}$ and $g(x) = x^5$

(c) $f(x) = \frac{1}{x}$ and $g(x) = (x+3)^5$

(d) (a) and (b)

(e) (a) and (c)

← answer

(a) $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x+3}\right) = \left(\frac{1}{x+3}\right)^5 = \frac{1^5}{(x+3)^5} = \frac{1}{(x+3)^5}$

(b) $(f \circ g)(x) = f(x^5) = \frac{1}{x^5+3}$

(c) $(f \circ g)(x) = f((x+3)^5) = \frac{1}{(x+3)^5}$

Section 2.1: Graphing Functions: Increasing, Decreasing

Some definitions:

Suppose that the function f is defined on an open interval I.

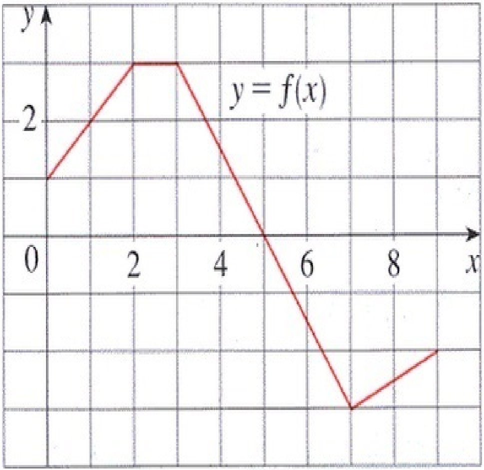
- f is *increasing* on I if for each a, b in I, if $a < b$, then $f(a) < f(b)$.
- f is *decreasing* on I if for each a, b in I, if $a < b$, then $f(a) > f(b)$.
- f is *constant* on I if $f(a) = f(b)$ for each a, b in I.

Note that going from left to right, the graph of f

- goes upward if f is increasing
- goes downward if f is decreasing
- is horizontal if f is constant.

Example

Identify the intervals (if any) on which f is increasing, decreasing, and constant.



The domain is $[0,9]$.

f is increasing on

$(0,2)$ and $(7,9)$

i.e. $(0,2) \cup (7,9)$

f is decreasing on

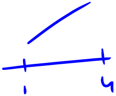
$(3,7)$

f is constant on $(2,3)$.

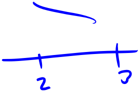
Question

Suppose the function $g(x)$ is **decreasing** on the interval $(0, 7)$. Which of the following is true?

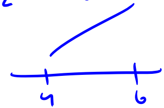
(a) $g(1) < g(4)$



(b) $g(2) > g(3)$



(c) $g(4) < g(6)$



(d) All of the above are true.

(e) None of the above are true.