# August 24 MATH 1113 sec. 52 Fall 2018

## Section 2.3: Compositions

Suppose a spherical balloon is inflated so that the radius after time $t$ seconds is given by the function $r(t) = 2t$ cm. The volume of a sphere of radius $r$ is known to be $V(r) = \frac{4}{3}\pi r^3$. Note that

- $r$ is a function of $t$, and
- $V$ is a function of $r$, making
- $V$ a function of $t$ (through its dependence on $r$). In fact,

$$V(t) = V(r(t)) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3.$$

This is an example of a **composition** of functions.

# Composition: Definition and Notation

Let $f$ and $g$ be functions. Then the **composite** function denoted

$$f \circ g,$$

also called the **composition** of $f$ and $g$, is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all $x$ in the domain of $g$ such that $g(x)$ is in the domain of $f$.

The expression $f \circ g$ is read "$f$ composed with $g$", and $(f \circ g)(x)$ is read "$f$ of $g$ of $x$".

## Example

Let $f(x) = \sqrt{x-1}$ and $g(x) = \frac{2}{x+1}$. Evaluate each expression if possible.

(a) $(f \circ g)(1)$ $= f(g(1)) = f\left(\frac{2}{1+1}\right) = f(1) = \sqrt{1-1} = 0$

(b) $(f \circ g)(0)$ $= f(g(0)) = f\left(\frac{2}{0+1}\right) = f(2) = \sqrt{2-1} = 1$

(c) $(g \circ f)(0)$ $= g(f(0)) = g\left(\sqrt{0-1}\right) = g\left(\sqrt{-1}\right)$

This is not defined.

0 is not in the domain of $g \circ f$ or of $f$

# Question

Let $f(x) = \sqrt{x-1}$ and $g(x) = \frac{2}{x+1}$. Evaluate $(g \circ f)(1)$ if possible.

(a) $(g \circ f)(1) = 0$

$= g(f(1)) = g(\sqrt{1-1})$

(b) $(g \circ f)(1) = 1$

$= g(0) = \dfrac{2}{0+1} = 2$

(c) $(g \circ f)(1) = 2$

(d) $(g \circ f)(1)$ is undefined

# Example $f(x) = \sqrt{x-1}$ and $g(x) = \frac{2}{x+1}$

Find a simplified formula for $F(x) = (f \circ g)(x)$ and determine its domain.

Note: The domain of $f$ is $[1, \infty)$.

* $f(x)$ is defined if $x-1 \geq 0$. $\Rightarrow$ $x \geq 1$

The domain of $g$ is $(-\infty, -1) \cup (-1, \infty)$.

* $g(x)$ is defined if we don't divide by zero.

so we require $x+1 \neq 0 \Rightarrow x \neq -1$.

$$F(x) = (f \circ g)(x) = f(g(x)) = f\left(\frac{2}{x+1}\right) = \sqrt{\frac{2}{x+1} - 1}$$

$$= \sqrt{\frac{2}{x+1} - \frac{x+1}{x+1}} = \sqrt{\frac{2-(x+1)}{x+1}} = \sqrt{\frac{2-x-1}{x+1}}$$

$$\boxed{F(x) = \sqrt{\frac{1-x}{x+1}}}$$

For $x$ in the domain of $F$ we require $x+1 \neq 0$ and $\frac{1-x}{x+1} \geq 0$.

So $x \neq -1$ and we need $\frac{2}{x+1} - 1 \geq 0$

If $x+1 \geq 0$ and $1-x \geq 0$

$x \geq -1$ and $x \leq 1$ with $x \neq -1$

we get $-1 < x \leq 1$. If $x+1 \leq 0$ and $1-x \leq 0$

Then $x \leq -1$ and $x \geq 1$ which has no solutions.

The domain of $F$ is the interval $(-1, 1]$.

## Example $f(x) = \sqrt{x-1}$ and $g(x) = \frac{2}{x+1}$

Find a simplified formula for $H(x) = (g \circ f)(x)$, and determine its domain.

$$H(x) = (g \circ f)(x) = g(f(x)) = g(\sqrt{x-1})$$

$$= \frac{2}{\sqrt{x-1} + 1}$$

For the domain, we need $x - 1 \geq 0 \Rightarrow x \geq 1$

We also require $\sqrt{x-1} + 1 \neq 0$.

If we consider the equation

$$\sqrt{x-1} + 1 = 0$$

then $\sqrt{x-1} = -1$

But $\sqrt{x-1} \geq 0$ so there are no solutions.

So the domain of $H$ is $[1, \infty)$.

# Question

The function $p(x) = \dfrac{1}{(x+3)^5}$ could be the composition $f \circ g$ of which pair of functions?

(a) $f(x) = x^5$, and $g(x) = \frac{1}{x+3}$ $f(g(x)) = f\left(\frac{1}{x+3}\right) = \left(\frac{1}{x+3}\right)^5 = \frac{1}{(x+3)^5}$

(b) $f(x) = \frac{1}{x+3}$ and $g(x) = x^5$ $f(g(x)) = \frac{1}{x^5+3}$

(c) $f(x) = \frac{1}{x}$ and $g(x) = (x+3)^5$ $f(g(x)) = \frac{1}{(x+3)^5}$

(d) (a) and (b)

(e) (a) and (c)

# Section 2.1: Graphing Functions: Increasing, Decreasing

**Some definitions:**

Suppose that the function $f$ is defined on an open interval $I$.

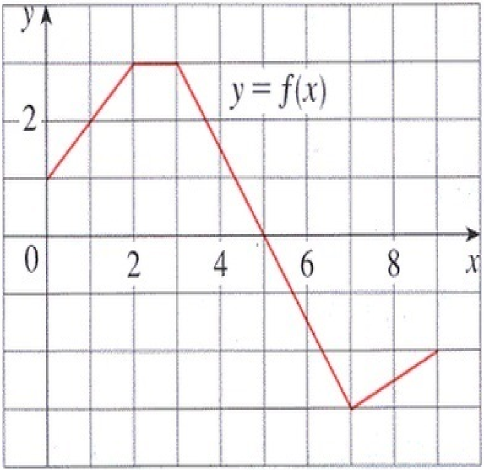
- $f$ is *increasing* on $I$ if for each $a$, $b$ in $I$, if $a < b$, then $f(a) < f(b)$.
- $f$ is *decreasing* on $I$ if for each $a$, $b$ in $I$, if $a < b$, then $f(a) > f(b)$.
- $f$ is *constant* on $I$ if $f(a) = f(b)$ for each $a$, $b$ in $I$.

Note that going from left to right, the graph of $f$

- goes upward if $f$ is increasing
- goes downward if $f$ is decreasing
- is horizontal if $f$ is constant.

# Example

Identify the intervals (if any) on which $f$ is increasing, decreasing, and constant.




$f$ is defined on $(0,9)$

$f$ is increasing on

$(0,2)$ and on $(7,9)$

i.e. $(0,2) \cup (7,9)$.

$f$ is decreasing

on $(3,7)$

$f$ is constant on $(2,3)$.