August 24 Math 1190 sec. 51 Fall 2016

Section 1.2: Limits of Functions Using Properties of Limits

Theorem: If $f(x) = A$ where A is a constant, then for any real number c

$$\lim_{x\to c} f(x) = \lim_{x\to c} A = A$$

Theorem: If $f(x) = x$, then for any real number c

$$\lim_{x\to c} f(x) = \lim_{x\to c} x = c$$

Additional Limit Law Theorems

Suppose

$$\lim_{x\to c} f(x) = L, \quad \lim_{x\to c} g(x) = M, \quad \text{and } k \text{ is constant.}$$

Theorem: (Sums) $\lim_{x\to c}(f(x)+g(x)) = L+M$

Theorem: (Differences) $\lim_{x\to c}(f(x)-g(x)) = L-M$

Theorem: (Constant Multiples) $\lim_{x\to c} kf(x) = kL$

Theorem: (Products) $\lim_{x\to c} f(x)g(x) = LM$

Examples

Use the limit law theorems to evaluate if possible

(a) $\lim_{x \to 2} (x^2 + 3x)$

we know $\lim_{x \to 2} x = 2$

$$\lim_{x \to 2} (x^2 + 3x) = \lim_{x \to 2} x^2 + \lim_{x \to 2} 3x$$

$$= \left(\lim_{x \to 2} x\right) \cdot \left(\lim_{x \to 2} x\right) + 3 \lim_{x \to 2} x$$

$$= 2 \cdot 2 + 3(2) = 10$$

Examples

Use the limit law theorems to evaluate if possible

(b) $\lim_{x\to 0} f(x)$ where $f(x) = \begin{cases} x+2, & x<0 \\ 1, & x=0 \\ 2x-3, & x>0 \end{cases}$

Let's look @ $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x+2) = \lim_{x\to 0^-} x + \lim_{x\to 0^-} 2$$

$$= 0 + 2 = 2$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (2x-3) =$$

$$= \lim_{x\to 0^+} 2x - \lim_{x\to 0^+} 3$$

$$= 2\lim_{x\to 0^+} x - \lim_{x\to 0^+} 3$$

$$= 2(0) - 3 \quad = -3$$

$\lim_{x\to 0^-} f(x) = 2$ and $\lim_{x\to 0^+} f(x) = -3$

so $\lim_{x\to 0} f(x)$ DNE (the graph must have a jump.)

Question

(1) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} x^2+1, & x \le 1 \\ 3-x, & x > 1 \end{cases}$

(a) 4

(b) 2

(c) 1

(d) DNE

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^2+1) = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (3-x) = 2$

so $\lim_{x\to 1} f(x) = 2$