August 24 Math 1190 sec. 52 Fall 2016

Section 1.2: Limits of Functions Using Properties of Limits

Theorem: If $f(x) = A$ where A is a constant, then for any real number c

$$\lim_{x \to c} f(x) = \lim_{x \to c} A = A$$

Theorem: If $f(x) = x$, then for any real number c

$$\lim_{x \to c} f(x) = \lim_{x \to c} x = c$$

Additional Limit Law Theorems

Suppose

$$\lim_{x\to c} f(x) = L, \quad \lim_{x\to c} g(x) = M, \quad \text{and } k \text{ is constant.}$$

Theorem: (Sums) $\lim_{x\to c}(f(x)+g(x)) = L+M$

Theorem: (Differences) $\lim_{x\to c}(f(x)-g(x)) = L-M$

Theorem: (Constant Multiples) $\lim_{x\to c} kf(x) = kL$

Theorem: (Products) $\lim_{x\to c} f(x)g(x) = LM$

Examples

Use the limit law theorems to evaluate if possible

(b) $\lim_{x \to -2} (x+1)^2$

Note $\lim_{x \to -2} (x+1) = \lim_{x \to -2} x + \lim_{x \to -2} 1$

$= -2 + 1 = -1$

so $\lim_{x \to -2} (x+1)^2 = \left(\lim_{x \to -2} (x+1) \right) \cdot \left(\lim_{x \to -2} (x+1) \right)$

$= (-1) \cdot (-1) = 1$

Examples

Use the limit law theorems to evaluate if possible

(c) $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} x+2, & x < 0 \\ 1, & x = 0 \\ 2x-3, & x > 0 \end{cases}$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x+2) = \lim_{x \to 0^-} x + \lim_{x \to 0^-} 2$$

$$= 0 + 2 = 2$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (2x-3) = 2 \lim_{x \to 0^+} x - \lim_{x \to 0^+} 3$$

$$= 2 \cdot 0 - 3 = -3$$

$$\lim_{x \to 0^+} f(x) = -3 \quad \text{and} \quad \lim_{x \to 0^-} f(x) = 2$$

$$\lim_{x \to 0} f(x) \text{ DNE}$$ (f has a jump)