August 27 MATH 1113 sec. 51 Fall 2018

Section 2.1: Graphing Functions: Increasing, Decreasing

Some definitions:
Suppose that the function f is defined on an open interval I.

- f is *increasing* on I if for each a, b in I, if $a < b$, then $f(a) < f(b)$.
- f is *decreasing* on I if for each a, b in I, if $a < b$, then $f(a) > f(b)$.
- f is *constant* on I if $f(a) = f(b)$ for each a, b in I.

Note that going from left to right, the graph of f

- goes upward if f is increasing
- goes downward if f is decreasing
- is horizontal if f is constant.

Question

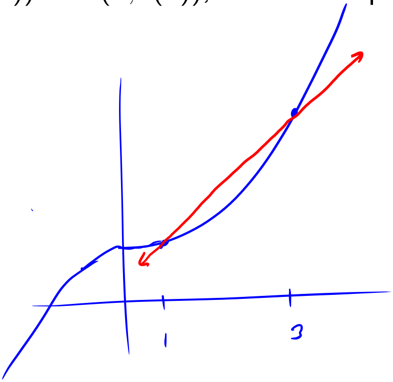
Suppose f is a function that is increasing on $(-\infty, \infty)$. If we draw a straight line through the points $(1, f(1))$ and $(3, f(3))$, then the slope of this line would be

(a) positive, and I'm confident

(b) positive, but I'm not sure

(c) negative, and I'm confident

(d) negative, but I'm not sure



(e) there's not enough information to determine whether the slope would be positive, negative, or zero

Relative Extrema

Some defintions:

Suppose f is a function and c is in the interior of the domain of f. Then ← not an end point

- $f(c)$ ← a y-value is a **relative maximum** if there exists an open interval I containing c such that $f(x) < f(c)$ for all x in I different from c,
- $f(c)$ ← a y-value is a **relative minimum** if there exists an open interval I containing c such that $f(x) > f(c)$ for all x in I different from c.

An **extremum** is a maximum or a minimum. The plurals of these three terms are extrema, maxima, and minima. The word *relative* can be replaced with the word **local**.

Relative Extrema

Relative extrema are the y-values for local highest or lowest points on a graph.

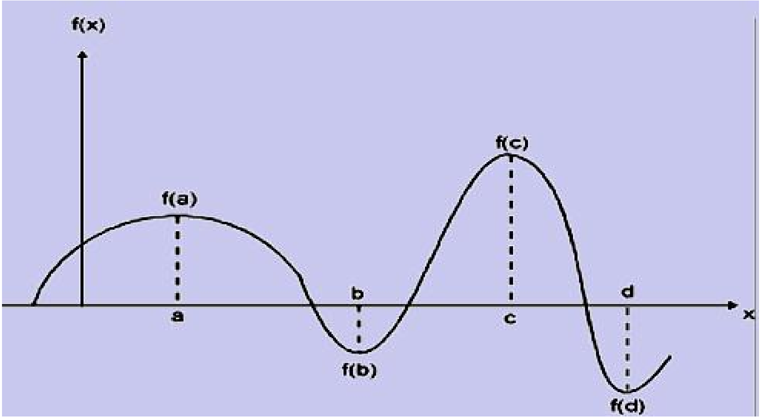


Figure: f has relative maxima $f(a)$ and $f(c)$ and relative minima $f(b)$ and $f(d)$

Minute Exercise

Draw the graph of a function f with domain $[0, 5]$ having the following properties:

- $f(0) = 1$ and $f(5) = 5$
- f is decreasing on $(0, 2)$, increasing on $(2, 4)$, and decreasing on $(4, 5)$
- f has relative minimum 0 when $x = 2$ and relative maximum 7 when $x = 4$.

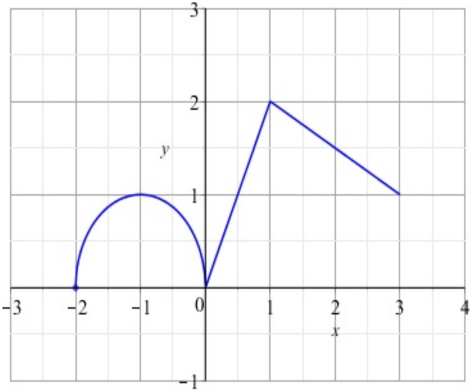
Section 2.5: Basic Transformations

From a small library of known function plots, we can graph a variety of functions if they can be determined as simple tranformations. We'll consider the following transformations:

- **Translations** shifting a graph up or down (vertical) or to the left or right (horizontal)
- **Reflections** taking the *mirror* image in the x or y axis
- **Scaling** stretching or shriking a graph in either of the vertical or horizontal orientations

Vertical Translation: $y = f(x) + b$ or $y = f(x) - b$

here $b > 0$



x	$f(x)$
-2	0
-1	1
0	0
1	2
2	$\frac{3}{2}$
3	1

Figure: The graph of $y = f(x)$ is shown along with a table of select points. Let's consider the plots of $y = f(x) + 1$ and $y = f(x) - 1$.

Vertical Translation: $y = f(x) + b$ or $y = f(x) - b$

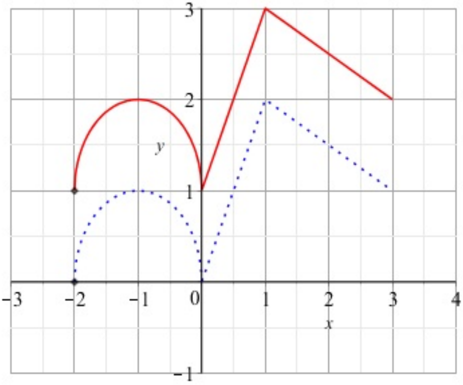
x	$f(x)$
-2	0
-1	1
0	0
1	2
2	$\frac{3}{2}$
3	1

x	$f(x)+1$
-2	$0+1=1$
-1	$1+1=2$
0	1
1	3
2	$5/2$
3	2

x	$f(x)-1$
-2	$0-1=-1$
-1	$1-1=0$
0	-1
1	1
2	$\frac{1}{2}$
3	0

Figure: Complete the tables of values.

Vertical Translation: $y = f(x) + b$ or $y = f(x) - b$



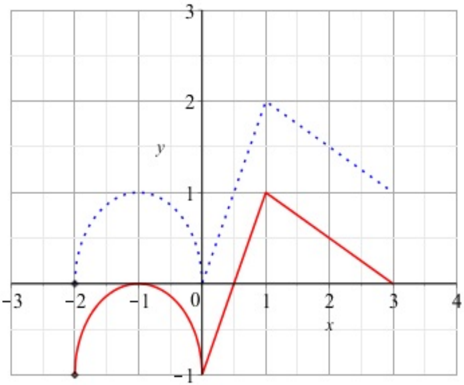
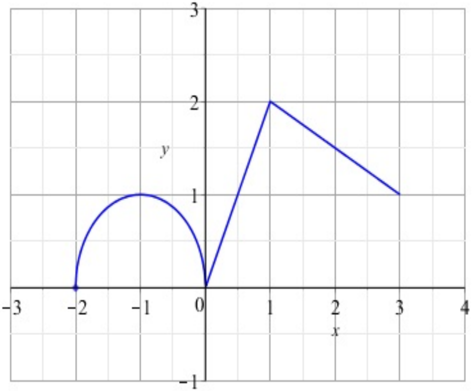


Figure: Left: $y = f(x)$ (blue dots), compared to $y = f(x) + 1$ (red)
Right: $y = f(x)$ (blue dots), compared to $y = f(x) - 1$ (red)

Horizontal Translation: $y = f(x - d)$ or $y = f(x + d)$



x	$f(x)$
-2	0
-1	1
0	0
1	2
2	$\frac{3}{2}$
3	1

Figure: The graph of $y = f(x)$ is shown along with a table of select points. Let's consider the plots of $y = f(x - 1)$ and $y = f(x + 1)$.

Horizontal Translation: $y = f(x - d)$ or $y = f(x + d)$

x	$f(x)$
-3	undef.
-2	0
-1	1
0	0
1	2
2	$\frac{3}{2}$
3	1
4	undef.

x	$f(x-1)$
-3	f(-4) - undef
-2	f(-3) - undef
-1	f(-2) = 0
0	1
1	0
2	2
3	$\frac{3}{2}$
4	1

x	$f(x+1)$
-3	f(-2) = 0
-2	f(-1) = 1
-1	0
0	2
1	3/2
2	1
3	undef
4	undef.

Figure: Complete the tables of values.

Horizontal Translation: $y = f(x - d)$ or $y = f(x + d)$

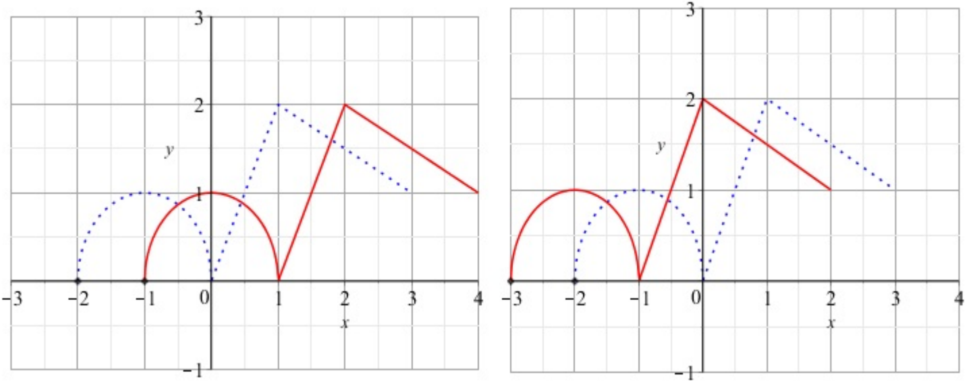


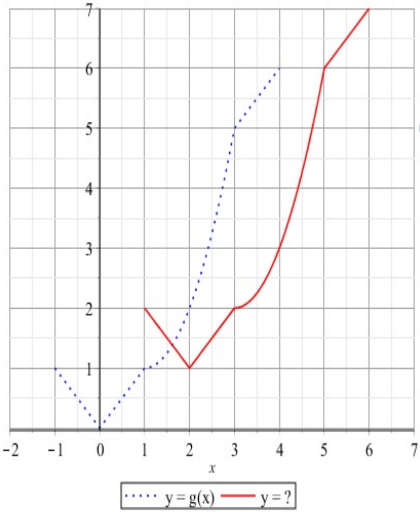
Figure: Left: $y = f(x)$ (blue dots), compared to $y = f(x - 1)$ (red)
Right: $y = f(x)$ (blue dots), compared to $y = f(x + 1)$ (red)

Vertical and Horizontal Translations

For $b > 0$ and $d > 0$

- the graph of $y = f(x) + b$ is the graph of $y = f(x)$ shifted up b units,
- the graph of $y = f(x) - b$ is the graph of $y = f(x)$ shifted down b units,
- the graph of $y = f(x - d)$ is the graph of $y = f(x)$ shifted right d units,
- the graph of $y = f(x + d)$ is the graph of $y = f(x)$ shifted left d units,

Question



The blue dotted curve is y = g(x). The red solid curve is the graph of y =

(a) $g(x-2)+1$

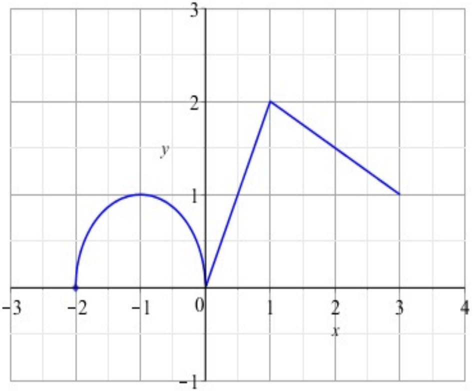
(b) $g(x+2)+1$

(c) $g(x-2)-1$

(d) $g(x+2)-1$

(e) can't be determined without more information

Reflections: $y = f(-x)$ or $y = -f(x)$



x	$f(x)$
-2	0
-1	1
0	0
1	2
2	$\frac{3}{2}$
3	1

Figure: The graph of $y = f(x)$ is shown along with a table of select points.
Now let's consider graphing $y = f(-x)$ and $y = -f(x)$

Reflections: $y = f(-x)$ or $y = -f(x)$

x	$f(x)$
-3	undef.
-2	0
-1	1
0	0
1	2
2	$\frac{3}{2}$
3	1

x	$f(-x)$
-3	$f(3)=1$
-2	$f(2)=3/2$
-1	2
0	0
1	1
2	0
3	undef

x	$-f(x)$
-3	undef
-2	$-f(-2)=-0=0$
-1	$-f(-1)=-1$
0	0
1	-2
2	$\frac{-3}{2}$
3	-1

Figure: Complete the tables of values.

Reflections: $y = f(-x)$ or $y = -f(x)$

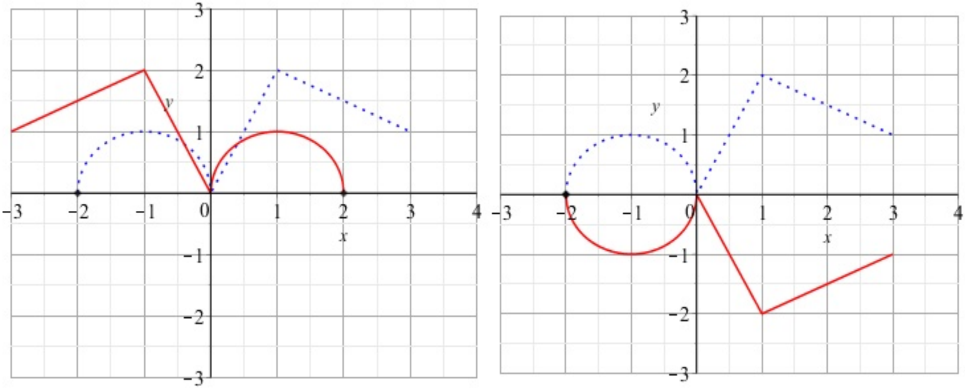


Figure: Left: $y = f(x)$ (blue dots), compared to $y = f(-x)$ (red)
Right: $y = f(x)$ (blue dots), compared to $y = -f(x)$ (red)

Reflection in the coordinate axes

The graph of $y = f(-x)$ is the reflection of the graph of $y = f(x)$ across the y-axis.

The graph of $y = -f(x)$ is the reflection of the graph of $y = f(x)$ across the x-axis.

Note that if (a, b) is a point on the graph of $y = f(x)$, then

(1) the point $(-a, b)$ is on the graph of $y = f(-x)$, and

(2) the point $(a, -b)$ is on the graph of $y = -f(x)$.

Stretching and Shrinking

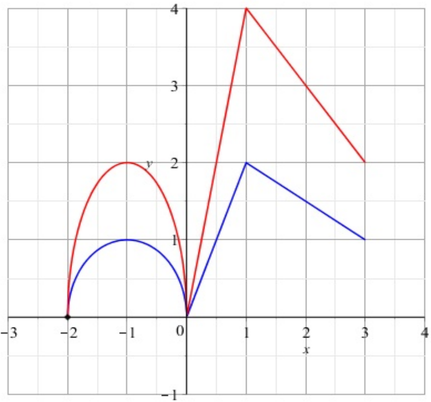
Since we already know that introducing a minus sign as in $f(-x)$ and $-f(x)$ results in a reflection, let's consider a positive number a and investigate the relationship between the graph of $y = f(x)$ and each of

$$y = af(x), \quad \text{and} \quad y = f(ax).$$

The outcome depends on whether $a > 1$ or $0 < a < 1$.

Why aren't we bothering with the case $a = 1$?

Vertical Stretch or Shrink: $y = af(x)$

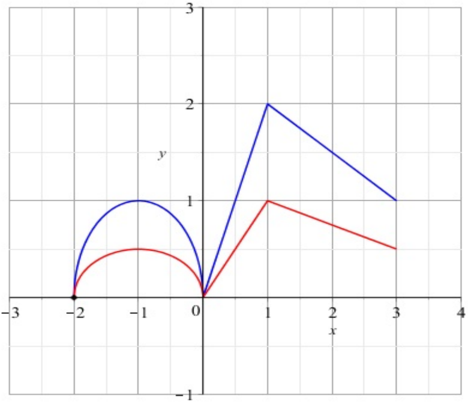


x	$f(x)$
-2	0
-1	1
0	0
1	2
2	$\frac{3}{2}$
3	1

x	$2f(x)$
-2	$2\cdot 0 = 0$
-1	$2\cdot 1 = 2$
0	0
1	4
2	3
3	2

Figure: $y = f(x)$ is in blue, and $y = 2f(x)$ is in red. Since $a = 2 > 1$, the graph is stretched vertically.

Vertical Stretch or Shrink: $y = af(x)$



x	$f(x)$
-2	0
-1	1
0	0
1	2
2	$\frac{3}{2}$
3	1

x	$\frac{1}{2}f(x)$
-2	$\frac{1}{2} \cdot 0 = 0$
-1	$\frac{1}{2} \cdot 1 = \frac{1}{2}$
0	0
1	1
2	$\frac{3}{4}$
3	$\frac{1}{2}$

Figure: $y = f(x)$ is in blue, and $y = \frac{1}{2}f(x)$ is in red. Since $a = \frac{1}{2} < 1$, the graph is shrinked vertically.

Vertical Stretch or Shrink: $y = af(x)$

The graph of $y = af(x)$ is obtained from the graph of $y = f(x)$. If $a > 0$, then

$y = af(x)$ is stretched vertically if $a > 1$, and
$y = af(x)$ is shrunk (a.k.a. compressed) vertically if $0 < a < 1$.

If $a < 0$, then the stretch ($|a| > 1$) or shrink ($0 < |a| < 1$) is combined with a reflection in the x-axis.

Horizontal Stretch or Shrink: $y = f(cx)$

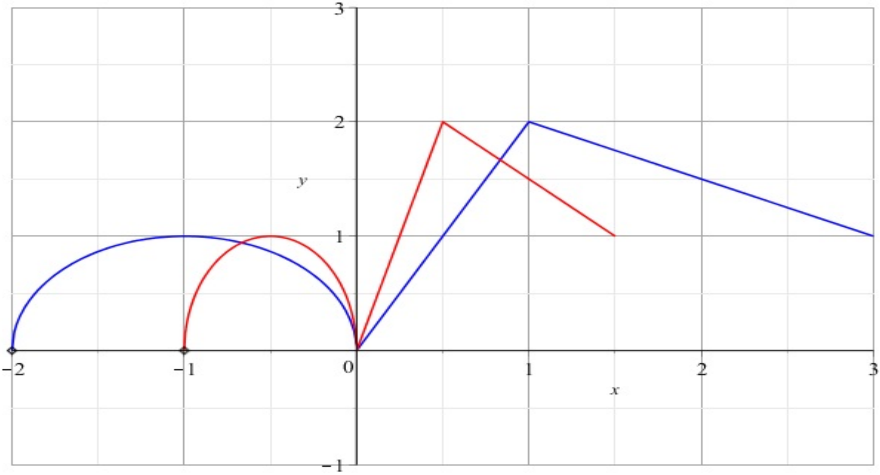


Figure: $y = f(x)$ is in blue, and $y = f(2x)$ is in red. Since $c = 2 > 1$, the graph is shrinked horizontally.

Horizontal Stretch or Shrink: $y = f(cx)$

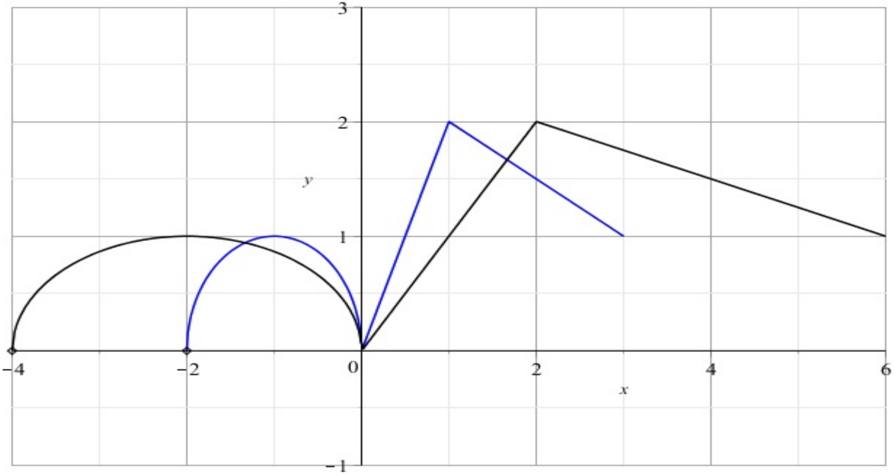


Figure: $y = f(x)$ is in blue, and $y = f\left(\frac{1}{2}x\right)$ is in black. Since $c = \frac{1}{2} < 1$, the graph is stretched horizontally.

Horizontal Stretch or Shrink: $y = f(cx)$

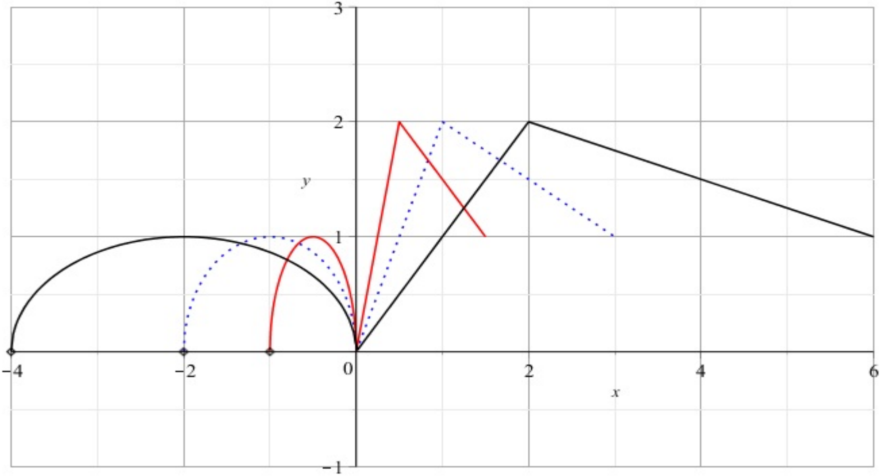


Figure: $y = f(x)$ is in blue dots. The compressed red curve is $y = f(2x)$, and the stretched black curve is $y = f\left(\frac{1}{2}x\right)$.

Horizontal Stretch or Shrink: $y = f(cx)$

The examples given generalize except that we did not consider an example with $c < 0$. This combines the stretch/shrink with a reflection. We have the following result:

The graph of $y = f(cx)$ is obtained from the graph of $y = f(x)$. If $c > 0$, then

$y = f(cx)$ is shrunk (a.k.a. compressed) horizontally if $c > 1$, and
$y = f(cx)$ is stretched horizontally if $0 < c < 1$.

If $c < 0$, then the shrink ($|c| > 1$) or stretch ($0 < |c| < 1$) is combined with a reflection in the y-axis.