# August 27 MATH 1113 sec. 52 Fall 2018

## Section 2.1: Graphing Functions: Increasing, Decreasing

**Some definitions:**

Suppose that the function $f$ is defined on an open interval $I$.

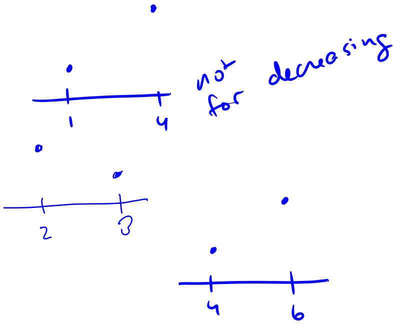
- $f$ is *increasing* on $I$ if for each $a$, $b$ in $I$, if $a < b$, then $f(a) < f(b)$.
- $f$ is *decreasing* on $I$ if for each $a$, $b$ in $I$, if $a < b$, then $f(a) > f(b)$.
- $f$ is *constant* on $I$ if $f(a) = f(b)$ for each $a$, $b$ in $I$.

Note that going from left to right, the graph of $f$

- goes upward if $f$ is increasing
- goes downward if $f$ is decreasing
- is horizontal if $f$ is constant.

# Question

Suppose the function $g(x)$ is **decreasing** on the interval $(0, 7)$. Which of the following is true?




(a) $g(1) < g(4)$

(b) $g(2) > g(3)$

(c) $g(4) < g(6)$

(d) All of the above are true.

(e) None of the above are true.

# Relative Extrema

**Some defintions:**

Suppose $f$ is a function and $c$ is in the interior of the domain of $f$. Then

- $f(c)$ is a **relative maximum** if there exists an open interval $I$ containing $c$ such that $f(x) < f(c)$ for all $x$ in $I$ different from $c$,
- $f(c)$ is a **relative minimum** if there exists an open interval $I$ containing $c$ such that $f(x) > f(c)$ for all $x$ in $I$ different from $c$.

An **extremum** is a maximum or a minimum. The plurals of these three terms are extrema, maxima, and minima. The word *relative* can be replaced with the word **local**.

# Relative Extrema

Relative extrema are the $y$-values for local highest or lowest points on a graph.

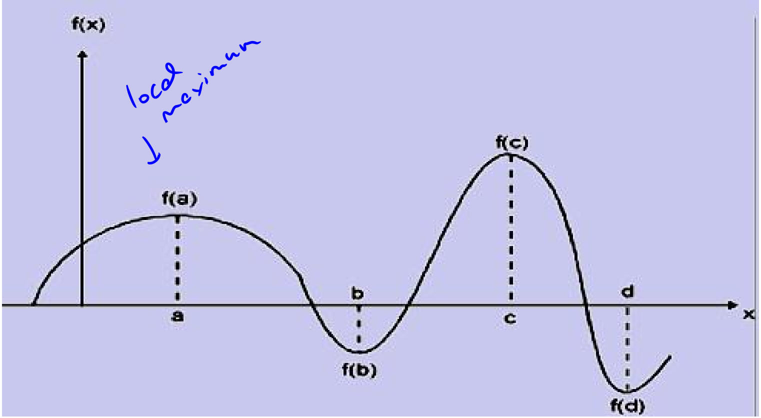



Figure: $f$ has relative maxima $f(a)$ and $f(c)$ and relative minima $f(b)$ and $f(d)$

# Minute Exercise

Draw the graph of a function $f$ with domain $[0,5]$ having the following properties:

- $f(0) = 1$ and $f(5) = 5$
- $f$ is decreasing on $(0,2)$, increasing on $(2,4)$, and decreasing on $(4,5)$
- $f$ has relative minimum 0 when $x = 2$ and relative maximum 7 when $x = 4$.

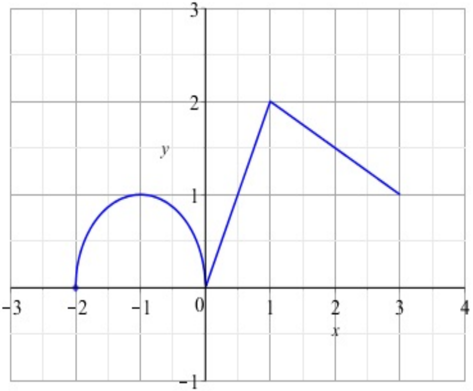
# Section 2.5: Basic Transformations

From a small library of known function plots, we can graph a variety of functions if they can be determined as simple tranformations. We'll consider the following transformations:

- **Translations** shifting a graph up or down (vertical) or to the left or right (horizontal)
- **Reflections** taking the *mirror* image in the $x$ or $y$ axis
- **Scaling** stretching or shriking a graph in either of the vertical or horizontal orientations

# Vertical Translation: $y = f(x) + b$ or $y = f(x) - b$

$b > 0$




| $x$ | $f(x)$ |
|---|---|
| $-2$ | $0$ |
| $-1$ | $1$ |
| $0$ | $0$ |
| $1$ | $2$ |
| $2$ | $\frac{3}{2}$ |
| $3$ | $1$ |

Figure: The graph of $y = f(x)$ is shown along with a table of select points. Let's consider the plots of $y = f(x) + 1$ and $y = f(x) - 1$.

# Vertical Translation: $y = f(x) + b$ or $y = f(x) - b$

| $x$ | $f(x)$ |
|---|---|
| $-2$ | $0$ |
| $-1$ | $1$ |
| $0$ | $0$ |
| $1$ | $2$ |
| $2$ | $\frac{3}{2}$ |
| $3$ | $1$ |

| $x$ | $f(x)+1$ |
|---|---|
| $-2$ | $0+1=1$ |
| $-1$ | $1+1=2$ |
| $0$ | $1$ |
| $1$ | $3$ |
| $2$ | $5/2$ |
| $3$ | $2$ |

| $x$ | $f(x)-1$ |
|---|---|
| $-2$ | $0-1=-1$ |
| $-1$ | $1-1=0$ |
| $0$ | $-1$ |
| $1$ | $1$ |
| $2$ | $\frac{1}{2}$ |
| $3$ | $0$ |

Figure: Complete the tables of values.

# Vertical Translation: $y = f(x) + b$ or $y = f(x) - b$

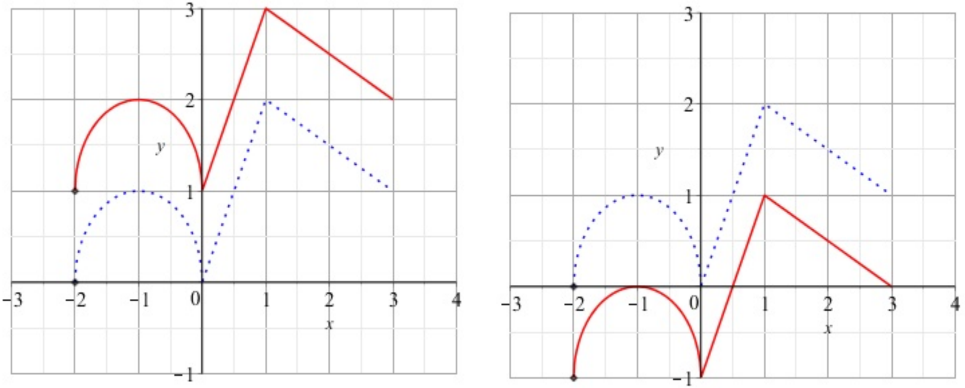
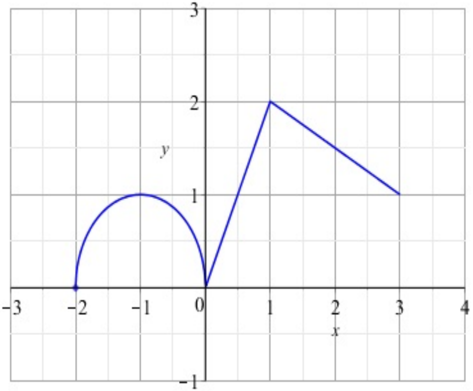



Figure: Left: $y = f(x)$ (blue dots), compared to $y = f(x) + 1$ (red)
Right: $y = f(x)$ (blue dots), compared to $y = f(x) - 1$ (red)

# Horizontal Translation: $y = f(x-d)$ or $y = f(x+d)$




| $x$ | $f(x)$ |
|---|---|
| $-2$ | 0 |
| $-1$ | 1 |
| 0 | 0 |
| 1 | 2 |
| 2 | $\frac{3}{2}$ |
| 3 | 1 |

Figure: The graph of $y = f(x)$ is shown along with a table of select points. Let's consider the plots of $y = f(x-1)$ and $y = f(x+1)$.

# Horizontal Translation: $y = f(x - d)$ or $y = f(x + d)$

| $x$ | $f(x)$ |
|---|---|
| $-3$ | undef. |
| $-2$ | 0 |
| $-1$ | 1 |
| 0 | 0 |
| 1 | 2 |
| 2 | $\frac{3}{2}$ |
| 3 | 1 |
| 4 | undef. |

| $x$ | $f(x-1)$ |
|---|---|
| $-3$ | f(-4) - undef |
| $-2$ | f(-3) - undef |
| $-1$ | f(-2) = 0 |
| 0 | 1 |
| 1 | 0 |
| 2 | 2 |
| 3 | $\frac{3}{2}$ |
| 4 | 1 |

| $x$ | $f(x+1)$ |
|---|---|
| $-3$ | f(-2) = 0 |
| $-2$ | f(-1) = 1 |
| $-1$ | 0 |
| 0 | 2 |
| 1 | $\frac{3}{2}$ |
| 2 | 1 |
| 3 | undef |
| 4 | undef |

Figure: Complete the tables of values.

# Horizontal Translation: $y = f(x - d)$ or $y = f(x + d)$

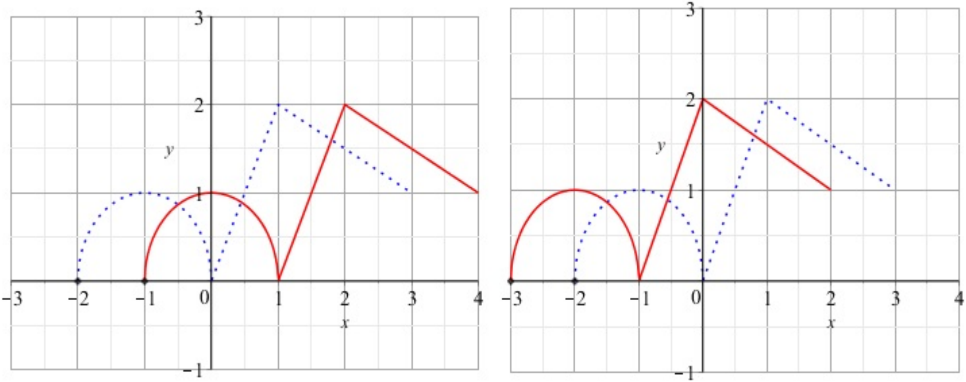


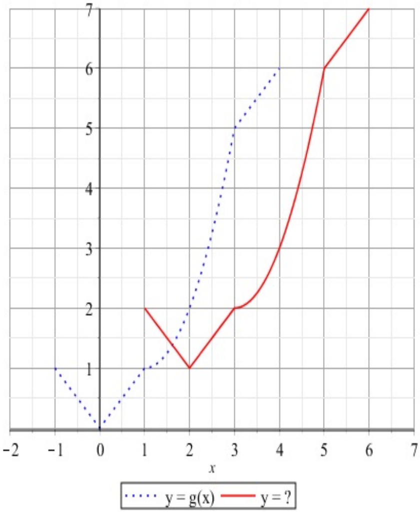

Figure: Left: $y = f(x)$ (blue dots), compared to $y = f(x - 1)$ (red)
Right: $y = f(x)$ (blue dots), compared to $y = f(x + 1)$ (red)

# Vertical and Horizontal Translations

For $b > 0$ and $d > 0$

- the graph of $y = f(x) + b$ is the graph of $y = f(x)$ shifted up $b$ units,
- the graph of $y = f(x) - b$ is the graph of $y = f(x)$ shifted down $b$ units,
- the graph of $y = f(x - d)$ is the graph of $y = f(x)$ shifted right $d$ units,
- the graph of $y = f(x + d)$ is the graph of $y = f(x)$ shifted left $d$ units,

# Question




The blue dotted curve is y = g(x). The red solid curve is the graph of y =

(a) $g(x-2)+1$

(b) $g(x+2)+1$

(c) $g(x-2)-1$

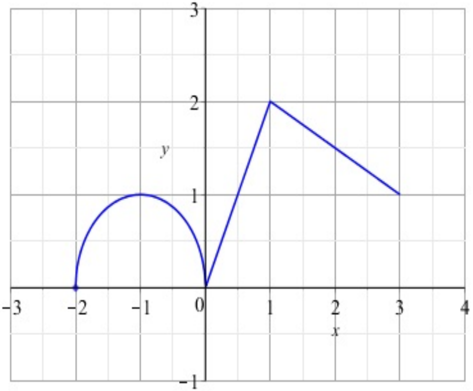
(d) $g(x+2)-1$

(e) can't be determined without more information

$x-2=0$

$x=2$

# Reflections: $y = f(-x)$ or $y = -f(x)$




| $x$ | $f(x)$ |
|---|---|
| $-2$ | $0$ |
| $-1$ | $1$ |
| $0$ | $0$ |
| $1$ | $2$ |
| $2$ | $\frac{3}{2}$ |
| $3$ | $1$ |

Figure: The graph of $y = f(x)$ is shown along with a table of select points.

Now let's consider graphing $y = f(-x)$ and $y = -f(x)$

# Reflections: $y = f(-x)$ or $y = -f(x)$

| $x$ | $f(x)$ |
|---|---|
| $-3$ | undef. |
| $-2$ | $0$ |
| $-1$ | $1$ |
| $0$ | $0$ |
| $1$ | $2$ |
| $2$ | $\frac{3}{2}$ |
| $3$ | $1$ |

| $x$ | $f(-x)$ |
|---|---|
| $-3$ | $f(3) = 1$ |
| $-2$ | $f(2) = \frac{3}{2}$ |
| $-1$ | $2$ |
| $0$ | $0$ |
| $1$ | $1$ |
| $2$ | $0$ |
| $3$ | undef |

| $x$ | $-f(x)$ |
|---|---|
| $-3$ | $-f(-3)$ undef |
| $-2$ | $-f(-2) = 0$ |
| $-1$ | $-f(-1) = -1$ |
| $0$ | $0$ |
| $1$ | $-2$ |
| $2$ | $-3/2$ |
| $3$ | $-1$ |

Figure: Complete the tables of values.

# Reflections: $y = f(-x)$ or $y = -f(x)$

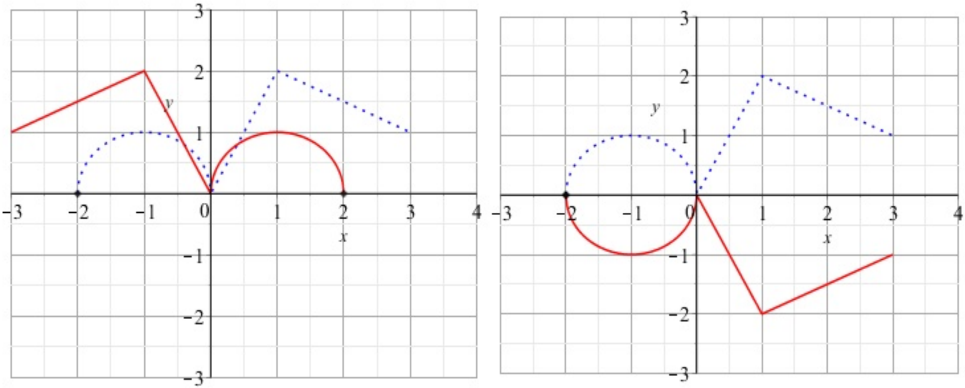



Figure: Left: $y = f(x)$ (blue dots), compared to $y = f(-x)$ (red)
Right: $y = f(x)$ (blue dots), compared to $y = -f(x)$ (red)