August 24 Math 2306 sec. 53 Fall 2018

Section 4: First Order Equations: Linear

- ► Put the equation in standard form y' + P(x)y = f(x), and correctly identify the function P(x).
- Obtain the integrating factor $\mu(x) = \exp(\int P(x) dx)$.
- Multiply both sides of the equation (in standard form) by the integrating factor µ. The left hand side will always collapse into the derivative of a product

$$\frac{d}{dx}[\mu(x)y] = \mu(x)f(x).$$

Integrate both sides, and solve for y.

$$y(x) = \frac{1}{\mu(x)} \int \mu(x) f(x) \, dx = e^{-\int P(x) \, dx} \left(\int e^{\int P(x) \, dx} f(x) \, dx + C \right)$$

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Solve the IVP

$$x \frac{dy}{dx} - y = 2x^{2}, x > 0 \quad y(1) = 5$$
Put in Standard form
(divide by, x)
$$\frac{dy}{dx} - \frac{1}{x} \quad y = 2x \qquad P(x) = \frac{-1}{x}$$
Integrating factor $\mu = e^{\int P(x) dx} = e^{\int \frac{-1}{x} dx} - 0nx \qquad \ln x^{1} - 1nx^{2}$

$$\sum_{x} \mu = x^{1} \qquad \text{mult. by } \mu$$

$$\sum_{x} \left(\frac{dy}{dx} - \frac{1}{x} y \right) = x^{1} (2x)$$

$$\frac{d}{dx} \left[x^{1} y \right] = 2$$

$$\int \frac{1}{dx} \left[\begin{array}{c} x' \\ y \end{array} \right] dx = \int 2 dx$$

$$x' \\ y = 2x^{2} + C$$

$$y = 2x^{2} + C$$

$$x \quad a \quad ore \quad ponometer \quad formity \\ ot \quad solutions \quad to \quad the \quad ODE$$

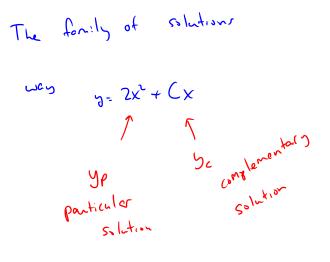
$$y = 2x^{2} + C$$

$$y = 2x^{2} + C$$

$$y = 2x^{2} + 3x$$

Now

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Verify

Just for giggles, lets verify that our solution $y = 2x^2 + 3x$ really does solve the differential equation we started with

$$x\frac{dy}{dx} - y = 2x^{2}.$$

$$y = 2x^{2} + 3x \quad s \quad y' = 4x + 3$$

$$x\frac{dy}{dx} - y = 2x^{2}.$$

$$x(4x + 3) - (2x^{2} + 3x) = 4x^{2} + 3x - 2x^{2} - 3x = 4x^{2} + 3x^{2} + 3x^{2} - 3x = 4x^{2} + 3x^{2} + 3x^{2$$

Steady and Transient States

For some linear equations, the term y_c decays as x (or t) grows. For example

$$\frac{dy}{dx} + y = 3xe^{-x}$$
 has solution $y = \frac{3}{2}x^2e^{-x} + Ce^{-x}$.
Here, $y_p = \frac{3}{2}x^2e^{-x}$ and $y_c = Ce^{-x}$.

Such a decaying complementary solution is called a transient state.

The corresponding particular solution is called a steady state.