Math 2335: Assignment 1
Review of Calculus Concepts

Instructions: Work collaboratively in groups of up to four. Please print each group members name legibly in the spaces provided. Complete as many problems with the highest degree of accuracy as you can during the class time. Use additional scratch paper as needed. Please record answers on this sheet and turn in at the end of class. Credit will be awarded based on evidence of effort. It is not necessary to finish all problems to receive full credit. Leaving early is not permitted and will result in a grade of 0% for this assignment. This assignment = one homework assignment.

(1) Evaluate each integral.

(a) \[ \int_0^1 (3x^3 - 4x + 2) \, dx = \frac{3}{4} \]

(b) \[ \int_1^4 \frac{1 + 6y^2}{y} \, dy = 3e - 1 \]
(c) $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

(d) $\int_{0}^{\frac{\pi}{4}} \sin^3 \theta \cos \theta \, d\theta = \frac{1}{4}$

(e) $\int xe^{2x} \, dx = \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C$
\[ \int 4te^{2} \, dx = 2e^2 + C \]

(2) Find the first derivative of each function.

(a) \( f(x) = \tan^{-1}(2x) \)

\[ f'(x) = \frac{2}{1 + 4x^2} \]

(b) \( g(x) = \cos^3(x^3 + 3x) \)

\[ g'(x) = -2(3x^2 + 3) \cos(x^3 + 3x) \sin(x^3 + 3x) \]
(c) \( f(x) = \frac{2x + 4}{x^4 + 3} \)

\[
f'(x) = \frac{6 - 6x^4 - 16x^3}{(x^4 + 3)^2}
\]

(d) \( h(t) = \ln(3t^2 - e^{2t}) \)

\[
h'(t) = \frac{(6t - 2e^{2t})}{3t^2 - e^{2t}}
\]

(3) Find the linearization of the given function centered at the specified point \( a \).

(a) \( f(x) = x \ln x \) at \( a = 1 \)

\[
L(x) = x - 1
\]
(b) \( f(x) = \sqrt{2} \cos x \) at \( a = \frac{\pi}{4} \)

\[
L(x) = 1 - (x - \frac{\pi}{4})
\]

(4) Find the Maclaurin series for the given function using any applicable method.

(a) \( f(x) = e^{3x} \)

\[
= \sum_{n=0}^{\infty} \frac{3^n x^n}{n!}
\]

\[
= 1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{6} + \ldots.
\]

(b) \( g(x) = \ln(1-x) \)

\[
= -\sum_{n=1}^{\infty} \frac{x^n}{n} = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots,
\]

Valid for \(-1 < x < 1\)
(5) Determine if each limit exists. If it exists, find its value. (It is possible that a given limit may be \( \infty \) or \(-\infty\).

(a) \( \lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9} = \frac{5}{6} \)

(b) \( \lim_{x \to -1} \frac{|x + 1|}{x + 1} \) \hspace{1cm} \text{Does not exist}

(c) \( \lim_{x \to 0} \ln(x^2) = -\infty \)
(d) \[ \lim_{x \to \infty} (\sqrt{x^2 + 4x} - x) = 0 \]

(c) \[ \lim_{x \to \frac{\pi}{2}^-} \tan(x) = \infty \]

(6) Find the solution set of each inequality. Express your answer in interval notation.

(a) \[ |x-4| < 0.5 \]

\[ (3.5, 4.5) \]
(b) $1 \leq |x+3| \quad (-\infty, -4] \cup [-2, \infty)$

(c) $|x+3| < 2 \quad (-5, -1)$

(d) $1 \leq |x+3| < 2 \quad (-5, -4] \cup [-2, -1)$

(e) $x \geq \frac{6}{x+1} \quad [-3, -1) \cup [2, \infty)$
(7) Locate and determine all relative extrema of the function \( f(x) = x^2e^{-x} \) — i.e. find all local maxima and minima and the \( x \)-values at which they occur.

\[ f \text{ has a local minimum } (0, 0) \]
\[ (\text{of } f \text{ at } x = 0) \]

and a local maximum \( (2, \frac{4}{e^2}) \)
\[ (\text{of } \frac{4}{e^2} \text{ at } x = 2) \]

(8) The graph of \( y = x^2e^{-x} \) has two points of inflection. Find the \( x \)-values at which they occur.

\[ @ \quad x = 2 + \sqrt{2} \quad \text{and} \quad x = 2 - \sqrt{2} \]
(9) The binomial coefficient \( \binom{\alpha}{k} \) is defined by
\[
\binom{\alpha}{k} = \frac{\alpha(\alpha - 1) \cdots (\alpha - k + 1)}{k!}, \quad k = 1, 2, 3, \ldots, \quad \binom{\alpha}{0} = 1.
\]
Evaluate each expression.

(a) \[ \binom{\frac{1}{2}}{2} = \frac{-1}{8} \]

(b) \[ \binom{\frac{3}{4}}{4} = \frac{-7}{243} \]

(10) Simplify each expression as much as possible.

(a) \[ \frac{n!}{(n+2)!} = \frac{1}{(n+1)(n+2)} \]

(b) \[ \frac{(2n+3)!}{(2n-1)!} = 2n(n+1)(2n+2)(2n+3) \]