(1) Given a nonzero vector $v$ in $\mathbb{R}^3$, we learned that $\text{Span}\{v\}$ is a line in the space that passed through the origin and is parallel to $v$. Let’s assume that $v$ is nonzero.

(a) Which of the following could be an alternative representation of $\text{Span}\{v\}$? Why/why not?

i. The set of all vectors $x$ in $\mathbb{R}^3$ such that $x = v$.

ii. The set of all vectors $x$ in $\mathbb{R}^3$ such that $x = tv$ for some $t$ in $\mathbb{R}$.

iii. The set of all vectors $x$ in $\mathbb{R}^3$ such that $x$ is parallel to $v$.

iv. The set of all vectors $x$ in $\mathbb{R}^3$ such that the set $\{x, v\}$ is linearly independent.

iv. The set of all vectors $x$ in $\mathbb{R}^3$ such that the set $\{x, v\}$ is linearly dependent.

(b) Suppose $x$ is a vector in $\text{Span}\{v\}$. Could $\text{Span}\{x, v\}$ be a plane in $\mathbb{R}^3$? Why/why not?

(c) Suppose $x$ is a vector in $\text{Span}\{v\}$. What are all possible geometric objects that $\text{Span}\{x, v\}$ could be (e.g. point, line, plane, 3d space)?

(d) Suppose $x$ is NOT in $\text{Span}\{v\}$. What are all possible geometric objects that $\text{Span}\{x, v\}$ could be (e.g. point, line, plane, 3d space)?
(2) Let \( A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}, \ b_1 = \begin{bmatrix} 7 \\ -2 \end{bmatrix}, \) and \( b_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}. \)

(a) Solve \( Ax = b_1 \) using your favorite technique. Call your solution \( u_1 \)

(b) Solve \( Ax = b_2 \) using your favorite technique. Call your solution \( u_2 \)

(c) Without actually solving, what would you conjecture is the solution of \( Ax = 3b_1 \)?

(d) Check to see if your conjecture is correct.

(e) Without actually solving, what would you conjecture is the solution of \( Ax = b_1 + b_2 \)?

(f) Check to see if your conjecture is correct.

(g) Without actually solving, what would you conjecture is the solution of \( Ax = 3b_1 - 2b_2 \)?

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This illustrates a property of linear transformations called the \textit{principle of superposition}. It gives us a way of breaking big problems into pieces and combining solutions we already have to get new ones.