Practice for Exam II MATH 2254H Spring 2015

Sections Covered: 7.1, 7.2, 7.3, 7.4, 7.5, 7.8, 10.1, 10.2

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.** Section 4.5 was traditional u-substitution, so it is included with the various integration techniques.

(1) Evaluate the given integrals using any applicable method.

(a) $\displaystyle\int x\sqrt{4x^2+6}\,dx$

(b) $\displaystyle\int_e^{e^2} \frac{dy}{y\ln y}$

(c) $\displaystyle\int \frac{dx}{x^2-1}$

(d) $\displaystyle\int \frac{x^3}{\sqrt{x^2+1}}\,dx$

(e) $\displaystyle\int \frac{x^2+7x+2}{(x^2+1)(x+3)}\,dx$

(f) $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx$

(g) $\displaystyle\int_0^{\frac{\pi}{4}} \tan^3 x\sec^3 x dx$

(h) $\displaystyle\int \sin^5 u\cos^3 u\,du$

(2) Use a substitution and then integration by parts to evaluate

$$\int e^{\sqrt{3s+9}}\,ds$$

(3) Evaluate the integral. (Hint, a substitution gives rise to a proper rational integrand.)

$$\int \frac{3\sin\theta\,d\theta}{\cos^2\theta+\cos\theta-2}$$

(4) Find the form of the partial fraction decomposition. (It is not necessary to find any of the coefficients A, B, etc.)

(a) $\dfrac{2x}{x^2 + 7x + 12}$

(b) $\dfrac{x^2 + 2x - 1}{(x^2 - 2x + 1)(x^2 - 4)}$

(c) $\dfrac{1}{(x + 2)^3(x^2 - 1)^2(x^2 + 4)^3}$

(5) Write the following as the sum of a polynomial and a proper rational function. Find a partial fraction decomposition for the resulting proper rational function.

$$\frac{x^4 + x^3 + 9x^2 + 8x - 11}{(x + 1)(x^2 + 9)}$$

(6) Determine if the improper integral is convergent or divergent. Evaluate it if possible. Note: You may be able to justify concluding that an integral is convergent or divergent even if it is not possible to evaluate it.

(a) $\displaystyle\int_0^1 \frac{\ln x}{x}\,dx$

(b) $\displaystyle\int_1^\infty \frac{(\tan^{-1} x)^2}{x^2 + 1}\,dx$

(c) $\displaystyle\int_2^\infty \frac{dx}{x^2 - 1}$

(d) $\displaystyle\int_{-1}^1 \cot x\,dx$

(e) $\displaystyle\int_1^\infty \frac{\tan^{-1} x}{x^2}\,dx$

(7) Find a Cartesian representation for the parametric curve. Give a rough sketch of the curve using arrows to show the orientation.

(a) $x = \cos(2t), \quad y = \cos t, \quad 0 \leq t \leq \dfrac{\pi}{2}$

(b) $x = \csc t, \quad y = \cot t, \quad 0 < t \leq \dfrac{\pi}{2}$

(c) $x = e^t, \quad y = e^{3t}, \quad -\infty < t \leq 0$

(8) Find a parameterization of the straight line segment that starts at the point $(0, 2)$ and ends at the point $(4, 3)$.

(9) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For which values of t is the curve concave upward.

$$x = t^2 + 1, \quad y = e^t - 1$$

(10) Find the points on the curve where the tangent is horizontal or vertical. $x = \sin\theta$, $y = \sin(2\theta)$.

(11) Find the area enclosed between the x-axis and the curve $x = e^t + 1$, $y = t - t^2$.

(12) Find the exact length of the curve $x = e^t + e^{-t}$, $y = 5 - 2t$, $0 \leq t \leq 3$.