Practice for Exam III MATH 2254H Spring 2015

Sections Covered: 10.1, 10.2, 10.3, 10.4, 11.1, 11.2, 11.3

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Eliminate the parameter to find a Cartesian representation for the curve defined by the parametric equations.

(a) $x = \ln t - 2, \quad y = t^2$

(b) $x = \cos\theta, \quad y = \cos(2\theta)$

(2) Find a set of parametric equations that defines the path that is the top half of the unit circle traversed from the point $(-1, 0)$ to the point $(1, 0)$.

(2) Find the equation of the line tangent to the parametric curve at the indicated point.

(a) $x = \ln t - 2, \quad y = t^2; \quad (-2, 1)$

(b) $x = \sec t, \quad y = \csc t, \quad$ at $\quad t = \dfrac{\pi}{4}$

(3) Convert each point from Cartesian to polar or vice versa.

(a) $(2, -2)$ to polar

(b) $(0, 3)$ to polar

(c) $\left(4, \frac{\pi}{3}\right)$ to Cartesian

(d) $\left(-1, \frac{7\pi}{6}\right)$ to Cartesian

(e) $(-3, 3)$ to polar

(f) $(2, 0)$ to Cartesian

(4) Find a Cartesian representation for the curve defined by the polar equation.

(a) $r = 2\csc\theta$ (b) $r^2\cos(2\theta) = 1$

(5) Find the area bounded by the given polar curve on the indicated interval.

(a) $r = 2\cos\theta, \quad 0 \le \theta \le \dfrac{\pi}{4}$

(b) $r = e^{-\theta}, \quad 0 \le \theta \le \pi$

(6) Plot the curve defined by each polar equation.

(a) $r = 4\sin\theta$ (b) $r = 1 + \cos\theta$

(c) $r = \sin(3\theta)$ (d) $r = 2\cos(2\theta)$

(7) Each of the following series is telescoping. Determine the convergence or divergence of the series. If convergent, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n^2 + 3n + 2}$

(8) Determine if the series converges or diverges. If convergent, find its sum.

(a) $\displaystyle\sum_{n=0}^{\infty} 6^{-n+1}3^{n-2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^n}{e^{2n-1}}$

(9) Determine the value(s) of x that solve the equation.

(a) $$\sum_{n=0}^{\infty} \frac{x^n}{4^n} = 5$$

(b) $$\sum_{n=0}^{\infty} \frac{1}{x^{2n}} = e$$

(10) Determine whether the integral test can be used to analyze the convergence or divergence of each series. If not, briefly explain why not. If so, use the test to determine if the series converges or diverges.

(a) $$\sum_{n=1}^{\infty} \frac{n}{n^4+1}$$

(b) $$\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$$

(c) $$\sum_{n=2}^{\infty} \frac{n}{n^2-2}$$

(d) $$\sum_{n=0}^{\infty} \frac{2^n}{n!}$$