Practice for Exam IV MATH 2254H Spring 2015

Sections Covered: 11.2–11.9

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Determine if the given series converges absolutely, converges conditionally, or diverges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt[3]{n+1}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^n}{(2n)!}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$

(d) $\displaystyle\sum_{n=1}^{\infty} a_n,$ where $a_1 = -3$ and $a_{n+1} = \frac{\ln n}{2^n} a_n$

(e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ Hint: root test

(f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \tan\left(\frac{1}{n}\right)$

(2) Find the radius and interval of convergence for the power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n} x^n}{3^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-3)^n}{4^n n!}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{(x-7)^n}{\ln(n)}$

(3) Find a power series whose interval of convergence is

(a) $(1, 5)$, (b) $[1, 5)$

(c) $(1, 5]$, (d) $[1, 5]$

(4) If k is a positive integer, find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n$$

(5) Find a power series representation for each function in powers of x (i.e. centered at zero). Identify the radius of convergence.

(a) $f(x) = \dfrac{x}{3 - x}$

(b) $g(x) = \ln(2+x)$, Hint: $g(0) = \ln(2)$ and $g'(x) = \dfrac{1}{2 + x}$.

(c) $h(x) = \dfrac{1}{(3 - x)^2}$, Hint: Look at $f'(x)$ from part (a).

(6) Given the power series representation $\tan^{-1}(x) = \sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ for $|x| < 1$, find a power series representation for the indefinite integral

$$\int \frac{\tan^{-1}(x)}{x}\, dx, \quad \text{for} \quad 0 < x < 1.$$