Exam I Review Questions: Math 2335 Spring 2014

Sections Covered: 1.1, 1.2, 2.2, 2.3, 3.1, 3.2

(1) (1.1 & 1.2) Write the Taylor polynomial with the remainder term for

(a) \( f(x) = \sqrt{x} \) centered at \( a = 4 \), of order 3.

(b) \( f(x) = \tan^{-1}(x) \) centered at \( a = 1 \), of order 1.

(2) (2.2) Rearrange the function being evaluated to avoid loss of significance errors. Use algebra and function identities as needed.

(a) \( f(x) = \ln(2 + x) - \ln(x) \) \( x \) very large

(b) \( f(x) = \sqrt[3]{8 + x^2} - 2 \) \( x \approx 0 \)

(c) \( f(x) = \frac{\sqrt{x^2 + 24} - 5}{x - 1} \) \( x \approx 1 \)

(3) (1.2 & 2.3) Use the result of (1 a) to bound the error when \( p_3(x) \) is used to approximate \( \sqrt{x} \) for \( 3.9 \leq x \leq 4.1 \).

(4) (2.3) Find the error and the relative error in the approximations \( x_A \approx x_T \).

(a) \( x_T = \sqrt{2}, \ x_A = 1.414 \) \quad (b) \( x_T = \ln 2, \ x_A = 0.7 \)

(5) (2.3, This is from problem #5 pg. 62.) In the following function evaluations \( f(x_A) \), assume that the numbers \( x_A \) are correctly rounded to the number of digits shown. Bound the error \( f(x_T) - f(x_A) \) and the relative error in \( f(x_A) \).

(a) \( \cos(1.473) \), \quad (b) \( e^{2.653} \)
(6) (3.1, Problem #11 pg. 78) Let $\alpha$ be the smallest positive root of $f(x) = 1 - x + \sin x$. Find an interval $[a, b]$ containing $\alpha$. Estimate the number of iterations needed to find $\alpha$ using the bisection method within an accuracy of $10^{-8}$.

(7) (3.2) Set up Newton’s method for finding the smallest positive root of the equation $\cos(x) = \sin(x)$. Use an initial guess of $x_0 = 0.5$ and compute the next two iterates $x_1$ and $x_2$ (give six decimal digits).

(8) (3.2) Give Newton’s method for finding $\sqrt[m]{a}$. That is, define an appropriate function whose root would be $\sqrt[m]{a}$, and set up the Newton’s iteration.