Exam I Review Questions: Math 2335 (Ritter)

Sections Covered: 1.1, 1.2, 2.2, 2.3, 3.1, 3.2, 3.3

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) (1.1 & 1.2) Write the Taylor polynomial with the remaider term for

(a) $f(x) = \sqrt{x}$ centered at $a = 4,$ of order $3.$

(b) $f(x) = \tan^{-1}(x)$ centered at $a = 1,$ of order $1.$

(2) (2.2) Rearrange the function being evaluated to aviod loss of significance errors. Use algebra and function identities as needed.

(a) $f(x) = \ln(2+x) - \ln(x)$ x very large

(b) $f(x) = \sqrt[3]{8+x^2} - 2$ $x \approx 0$

(c) $f(x) = \cos(\alpha) - \cos(\alpha + x)$ $x \approx 0$

(3) (2.3) Find the error and the relative error in the approximations $x_A \approx x_T$.

$$\text{(a)} \quad x_T = \sqrt{2},\ x_A = 1.414, \quad \text{(b)} \quad x_T = \ln 2,\ x_A = 0.7$$

(4) (2.3, This is from problem #5 pg. 62.) In the following function evaluations $f(x_A)$, assume that the numbers x_A are correctly rounded to the number of digits shown. Bound the error $f(x_T) - f(x_A)$ and the relative error in $f(x_A)$.

$$\text{(a)} \quad \cos(1.473), \quad \text{(b)} \quad e^{2.653}$$

(5) (3.1, Problem #11 pg. 78) Let α be the smallest positive root of $f(x) = 1 - x + \sin x$. Find an interval $[a, b]$ containing α. Estimate the number of iterations needed to find α using the bisection method within an accuracy of 10^{-8}.

(6) (3.2 & 3.3) Give Newton's method for finding $\sqrt[m]{a}$. That is, define an appropriate function whose root would be $\sqrt[m]{a}$, and set up the Newton's iteration. What would the iteration be for the Secant method?

(7) (3.2) Use the inequality for convergence $|\alpha - x_0| < \left|\frac{2f'(\alpha)}{f''(\alpha)}\right|$ to find an interval for the initial guess x_0 so that Newton's method will converge for $f(x) = \sec^{-1} x - \frac{\pi}{3}$. Note, the root is $\alpha = 2$, and you can assume that $x > 0$.

(8) (3.3) Show that if the Secant method is used to try to find the root $\alpha = 0$ of $f(x) = \sqrt[3]{x}$, that

$$x_{n+1} = -\left(x_n^{2/3} x_{n-1}^{1/3} + x_n^{1/3} x_{n-1}^{2/3}\right).$$

Will the secant method converge for any choice of x_0 and x_1 both different from zero?

(9) (3.3) Construct an algorithm to approximate $\sqrt{7}$ using the secant method. That is, define an appropriate function, and determine the secant method iteration formula. Choose a pair of initial guesses x_0 and x_1 and find the next two terms x_2 and x_3 in a sequence of approximations.