Exam 1 Math 2254 sec. 001

Summer 2015

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
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10	

INSTRUCTIONS: There are 10 problems worth 10 points each. No use of notes, books, or calculator is allowed. Illicit use of notes, a book, calculator, or any smart device will result in a grade of zero on this exam and may result in a formal allegation of academic misconduct. Show all of your work on the paper provided to receive full credit.

(1) Find the most general antiderivative of each function.

(a) $f(x) = \sqrt[4]{x} + \dfrac{1}{\sqrt{x}}$ $= x^{1/4} + x^{-1/2}$

$$F(x) = \frac{x^{5/4}}{5/4} + \frac{x^{1/2}}{1/2} + C$$

$$= \frac{4}{5} x^{5/4} + 2\sqrt{x} + C$$

(b) $f(x) = \dfrac{2}{x} - \dfrac{1}{\sqrt{1-x^2}}$

$$F(x) = 2\ln|x| - \sin^{-1} x + C$$

(2) Evaluate each derivative using the Fundamental Theorem of Calculus (part 1).

(a) $\dfrac{d}{dx}\left[\displaystyle\int_1^{x^2} \ln(t+1)\,dt\right]$ $= \ln(x^2+1)(2x)$

$$= 2x\ln(x^2+1)$$

(b) $\dfrac{d}{dx}\left[\displaystyle\int_x^7 e^{-t^3}\,dt\right]$ $= \dfrac{d}{dt}\left[-\displaystyle\int_7^x e^{-t^3}\,dt\right] = -e^{-x^3}$

(3) Suppose

$$\int_0^1 f(x)\,dx = -2, \quad \int_0^5 f(x)\,dx = 4, \quad \int_0^1 g(x)\,dx = 1, \quad \text{and} \quad \int_0^5 g(x)\,dx = 7.$$

(a) Evaluate $\int_1^5 f(x)\,dx$ $= \int_0^5 f(x)\,dx - \int_0^1 f(x)\,dx = 4 - (-2) = 6$

(b) Evaluate $\int_0^1 [2f(x) - 3g(x)]\,dx$ $= 2\int_0^1 f(x)\,dx - 3\int_0^1 g(x)\,dx$

$= 2(-2) - 3(1)$

$= -4 - 3 = -7$

(4) Recall the identity $1 + \tan^2\theta = \sec^2\theta$. Evaluate the indefinite integral.

$\int \tan^2\theta\,d\theta$ $= \int (\sec^2\theta - 1)\,d\theta$

$= \int \sec^2\theta\,d\theta - \int d\theta$

$= \tan\theta - \theta + C$

(5) Evaluate the definite integral using any method.

$$\int_1^2 \frac{2x^2+1}{x}\,dx = \int_1^2 \frac{2x^2}{x}\,dx + \int_1^2 \frac{1}{x}\,dx$$

$$= \int_1^2 2x\,dx + \int_1^2 \frac{1}{x}\,dx$$

$$= x^2 \Big|_1^2 + \ln|x| \Big|_1^2$$

$$= 4 - 1 + \ln 2 - \ln 1$$

$$= 3 + \ln 2$$

(6) Evaluate the definite integral using any method.

$$\int_0^{\ln 2} e^x \sqrt{1+e^x}\,dx$$

Let $u = 1+e^x$ $\quad du = e^x\,dx$

when $x=0$, $u = 1+e^0 = 2$

$x = \ln 2$, $u = 1 + e^{\ln 2} = 1+2 = 3$

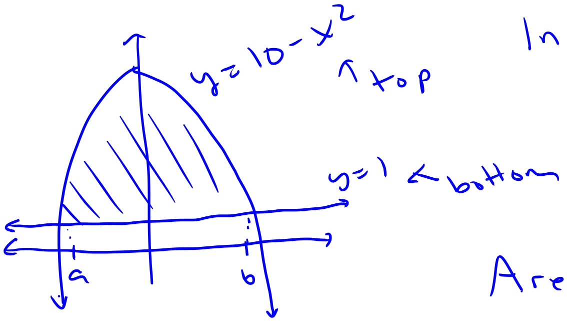
$$= \int_2^3 \sqrt{u}\,du$$

$$= \int_2^3 u^{1/2}\,du = \frac{u^{3/2}}{3/2}\Big|_2^3 = \frac{2}{3} u^{3/2}\Big|_2^3$$

$$= \frac{2}{3}(3)^{3/2} - \frac{2}{3}(2)^{3/2}$$

This doesn't simplify much.

(7) Find the area of the region bounded between the curves $y = 10 - x^2$ and $y = 1$.



Intersections: $10 - x^2 = 1 \Rightarrow 9 = x^2$
$\Rightarrow x = \pm 3$

$$\text{Area} = \int_{-3}^{3} (10 - x^2 - 1)\,dx$$

$$= \int_{-3}^{3} (9 - x^2)\,dx \qquad \text{using even symmetry}$$

$$= 2\int_{0}^{3} (9 - x^2)\,dx$$

$$= 2\left[9x - \frac{x^3}{3}\right]_0^3 = 2\left[9 \cdot 3 - \frac{3^3}{3}\right]$$

$$= 2\left[27 - 9\right] = 2(18) = 36$$

(8) Determine if each statement is *True* or *False*. Assume f is integrable, and indicate your conclusion with T or F in the blank provided.

(a) $\displaystyle\int_a^b 2f(x)\,dx = \left(\int_a^b 2\,dx\right)\left(\int_a^b f(x)\,dx\right)$ ___F___

(b) $\displaystyle\int_a^b (f(x) + 2)\,dx = \int_a^b f(x)\,dx + 2(b - a)$ ___T___

(c) $\displaystyle\int_a^b 2xf(x^2)\,dx = \int_{a^2}^{b^2} f(u)\,du$ ___T___

(9) Solve the differential equation subject to the given initial condition.

$$\frac{dy}{dt} = xe^{x^2}, \quad y(0) = 0$$

$y = \int x e^{x^2} dx$ Let $u = x^2$ $du = 2x\,dx \Rightarrow \frac{1}{2} du = x\,dx$

$= \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$

$y(x) = \frac{1}{2}e^{x^2} + C$, $y(0) = \frac{1}{2}e^{0^2} + C = 0$

$\frac{1}{2} + C = 0 \Rightarrow C = -\frac{1}{2}$

$$\boxed{y = \frac{1}{2}e^{x^2} - \frac{1}{2}}$$

(10) Evaluate the indefinite integral using the given substitution.

$\int \frac{x}{x+1}\,dx$, let $u = 1+x$ $du = dx$, $x = u-1$

$= \int \frac{u-1}{u}\,du$

$= \int \left(\frac{u}{u} - \frac{1}{u}\right) du = \int \left(1 - \frac{1}{u}\right) du$

$= u - \ln|u| + C$

$= x+1 - \ln|x+1| + C$