Exam 1 Math 2306 sec. 52

Summer 2016

Name: (4 points) Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	

INSTRUCTIONS: There are 6 problems worth 16 points each. You may use one sheet (8.5" $\times$ 11") of your own prepared notes/formulas.

No use of a calculator, text book, smart device, or other resource is permitted. Illicit use of any additional resource will result in a grade of zero on this exam as well as a formal allegation of academic misconduct. Show all of your work on the paper provided to receive full credit.

(1) Verify that for any constants c_1 and c_2 that $y = c_1 e^{-2x} + c_2 e^{4x} - 2$ is a solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 16$$

$y' = -2c_1 e^{-2x} + 4c_2 e^{4x}$

$y'' = 4c_1 e^{-2x} + 16 c_2 e^{4x}$

Substitute

$$y'' - 2y' - 8y \stackrel{?}{=} 16$$

$$4c_1 e^{-2x} + 16 c_2 e^{4x} - 2\left(-2c_1 e^{-2x} + 4c_2 e^{4x}\right) - 8\left(c_1 e^{-2x} + c_2 e^{4x} - 2\right) \stackrel{?}{=} 16$$

$$4c_1 e^{-2x} + 16c_2 e^{4x} + 4c_1 e^{-2x} - 8c_2 e^{4x} - 8c_1 e^{-2x} - 8c_2 e^{4x} + 16 \stackrel{?}{=} 16$$

$$c_1 e^{-2x} \underbrace{(4+4-8)}_{0} + c_2 e^{4x} \underbrace{(16-8-8)}_{0} + 16 \stackrel{?}{=} 16$$

$16 = 16$ an Identity

Hence $y = c_1 e^{-2x} + c_2 e^{4x} - 2$ solves the ODE for any c_1, c_2.

(2) Use the information from problem (1) to find the solution of the initial value problem.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 16, \qquad y(0) = 1, \quad y'(0) = 0$$

Using y' from (1).

$y(0) = c_1 e^0 + c_2 e^0 - 2 = c_1 + c_2 - 2 = 1$

$y'(0) = -2c_1 e^0 + 4c_2 e^0 = -2c_1 + 4c_2 = 0$

$c_1 + c_2 = 3$

$-2c_1 + 4c_2 = 0 \implies c_1 = 2c_2$

$3 = c_1 + c_2 = 2c_2 + c_2 = 3c_2 \implies c_2 = 1, \; c_1 = 2$

$$\boxed{\text{The solution to the IVP is } \quad y = 2e^{-2x} + e^{4x} - 2}$$

(3) Identify each equation as being Linear or Nonlinear. If an equation is nonlinear, identify at least one nonlinear term.

(a) $x^2y''+2xy'+4y = e^x$

This is linear.

(b) $x^2y''+2yy'+4y = e^x$

This is nonlinear. $2yy'$ is a nonlinear term.

(c) $\cos(t)\dfrac{dx}{dt}+\sin(t)x = \tan(x)$

This is nonlinear. $\tan x$ is a nonlinear term.

(4) Find all solutions of the separable equation. Implicit or explicit, your choice.

$$\frac{dy}{dx} = \frac{y^2+1}{x-2}$$

$$\frac{1}{y^2+1}\frac{dy}{dx} = \frac{1}{x-2}$$

$$\Rightarrow \frac{1}{y^2+1}\,dy = \frac{1}{x-2}\,dx$$

$$\int \frac{1}{y^2+1}\,dy = \int \frac{1}{x-2}\,dx$$

$$\boxed{\tan^{-1} y = \ln|x-2| + C}$$

All solns. are given implicitly by this.

(5) Find the general solution of the first order linear equation.

$$\frac{dy}{dx} + 2y = 6x^2 e^{-2x}$$

$P(x) = 2$ so $\mu = e^{\int P(x)dx} = e^{\int 2 dx} = e^{2x}$

$e^{2x}\left(\frac{dy}{dx} + 2y\right) = 6x^2 e^{-2x} \cdot e^{2x}$

$\frac{d}{dx}\left[e^{2x} y\right] = 6x^2$

$\int \frac{d}{dx}\left[e^{2x} y\right] dx = \int 6x^2 dx$

$e^{2x} y = 2x^3 + C$

$\boxed{y = 2x^3 e^{-2x} + C e^{-2x}}$

(6) Solve the initial value problem. (You may give an implicit or explicit solution, your choice.)

$$\frac{dx}{dt} = \sqrt{x}\,\sin(t), \quad x(0) = 1$$

Separate variables

$\frac{1}{\sqrt{x}} \frac{dx}{dt} = \sin t$

$\int x^{-1/2} dx = \int \sin t \, dt \Rightarrow \frac{x^{1/2}}{1/2} = -\cos t + C$

$2\sqrt{x} = -\cos t + C$ Apply $x(0) = 1$

$2\sqrt{1} = -\cos 0 + C \Rightarrow C = 2 + 1 = 3$

$\boxed{\text{Implicitly, the soln. to the IVP is } 2\sqrt{x} = 3 - \cos t}$