# Exam 1 Math 2306 sec. 53

Fall 2018

**Name:** (4 points) Solutions

Your signature (required) confirms that you agree to practice academic honesty.

**Signature:** ____________________

| Problem | Points |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

INSTRUCTIONS: There are 6 problems worth 16 points each. You may use one sheet (8.5" × 11") of your own prepared notes/formulas. **No use of a calculator, text book, smart device, or other resource is permitted. Illicit use of any additional resource will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** Show all of your work on the paper provided to receive full credit.

(1) Determine if each ODE is linear or nonlinear. If nonlinear, explain why (for example, identify at least one term in the equation that makes it nonlinear).

(a) $x^2\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+4y=e^x$ Linear

(b) $\dfrac{dy}{dx}=\sqrt{1-y^2}$ Nonlinear $y$ is dependent and we have $\sqrt{1-y^2}$

(c) $\left(\dfrac{d^2x}{dt^2}\right)^2+2x\dfrac{dx}{dt}=0$ nonlinear $x$ is dependent so $\left(\dfrac{d^2x}{dt^2}\right)^2$ is a nonlinear term

(d) $\cos(t)\dfrac{du}{dt}+\sin(t)\,u=1$ Linear

(2) Given that $y = c_1 e^x + c_2 e^{-2x}$ is a two parameter family of solutions of $y'' + y' - 2y = 0$, find the solution of the initial value problem

$$y'' + y' - 2y = 0, \quad y(0) = 2, \quad y'(0) = -7$$

$y = c_1 e^x + c_2 e^{-2x}$ $\qquad$ $y(0) = c_1 e^0 + c_2 e^0 = 2$

$y' = c_1 e^x - 2c_2 e^{-2x}$ $\qquad$ $y'(0) = c_1 e^0 - 2c_2 e^0 = -7$

$\Rightarrow$

$c_1 + c_2 = 2$

$c_1 - 2c_2 = -7$

$3c_2 = 9$

$c_2 = 3$

$c_1 = 2 - c_2 = -1$

The solution to the IVP is

$$\boxed{y = -e^x + 3e^{-2x}}$$

(3) Solve the initial value problem. (Your answer can be implicit or explicit, your choice.)

$$6\frac{dy}{dx} = \frac{xe^x}{y^2}, \quad y(0) = 1$$

separable

$6y^2 \frac{dy}{dx} = xe^x \quad \Rightarrow \quad \int 6y^2\, dy = \int xe^x\, dx$

$u = x \qquad du = dx$

$v = e^x \qquad dv = e^x$

$2y^3 = xe^x - e^x + C$

when $x = 0, y = 1$ $\qquad$ $2(1)^3 = 0e^0 - e^0 + C$

$2 = -1 + C \Rightarrow C = 3$

The solution is given implicitly by

$$\boxed{2y^3 = xe^x - e^x + 3}$$

(4) Solve the initial value problem. (Provide an explicit solution.)

$$\frac{dy}{dt} + 2y = e^{-2t}\cos t \qquad y(0) = -2$$

Linear $\quad P(t) = 2 \quad \mu = e^{\int P(t)dt} = e^{2t}$

$\frac{d}{dt}\left(e^{2t} y\right) = e^{2t} \cdot e^{-2t} \cos t = \cos t$

$\int \frac{d}{dx}\left(e^{2t} y\right) dt = \int \cos t \, dt$

$e^{2t} y = \sin t + C \quad \Rightarrow \quad y = e^{-2t}\sin t + C e^{-2t}$

$y(0) = e^{0}\sin(0) + C e^{0} = -2$

$C = -2$

The solution is

$\boxed{y = e^{-2t}\sin t - 2e^{-2t}}$

(5) For each first order differential equation, identify whether it is (I) Linear, (II) Separable, or (III) Bernoulli.

(a) $\frac{dy}{dx} = y - x \qquad y' - y = -x \qquad$ Linear (I)

(b) $\frac{dy}{dx} = y^2 - xy \qquad y' + xy = y^2 \qquad$ Bernoulli (III)

(c) $t^3 \frac{dy}{dt} = t\cos(y) \qquad y' = \frac{1}{t^2}\cos y \qquad$ Separable (II)

(d) $\frac{dx}{dt} + \frac{2}{t^2}x = t\sqrt{x} \qquad x' + \frac{2}{t^2}x = t x^{1/2} \qquad$ Bernoulli (III)

(6) Find the general solution of the Bernoulli equation $\dfrac{dy}{dx} + y = -e^x y^2$.

Here $n=2$ so let $u = y^{1-2} = y^{-1}$

$$u' = -y^{-2} y' \Rightarrow y' = -y^2 u'$$

$$-y^2 u' + y = -e^x y^2$$

$$u' - \frac{y}{y^2} = \frac{-e^x y^2}{-y^2} \Rightarrow u' - \underbrace{y^{-1}}_{u} = e^x$$

$u' - u = e^x$ $\quad P(x) = -1$ so $\mu = e^{\int P(x)dx} = e^{-x}$

$$\frac{d}{dx}\left(e^{-x} u\right) = e^{-x} \cdot e^{x} = 1$$

$$\int \frac{d}{dx}\left(e^{-x} u\right) dx = \int dx$$

$$e^{-x} u = x + C \Rightarrow u = xe^x + Ce^x$$

Finally, $y = u^{-1} = \frac{1}{u}$ so the solution

$$\boxed{y = \frac{1}{xe^x + Ce^x}}$$