Exam 1 Math 2306 sec. 59

Spring 2016

Name: (4 points) Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	

INSTRUCTIONS: There are 8 problems worth 12 points each. You may use one sheet (8.5" × 11") of your own prepared notes/formulas.

No use of a calculator, text book, smart device, or other resource is permitted. Illicit use of any additional resource will result in a grade of zero on this exam as well as a formal allegation of academic misconduct. Show all of your work on the paper provided to receive full credit.

(1) Determine if each ODE is linear or nonlinear. If nonlinear, explain why (for example, identify at least one term in the equation that makes it nonlinear).

(a) $y^4 + 2\dfrac{dy}{dt} = \cos t$ This is nonlinear. y^4 is a nonlinear term.

(b) $\sin y \dfrac{dy}{dx} - \cos x = y$ This is nonlinear. $\sin y$ and $\sin y \dfrac{dy}{dx}$ are both nonlinear.

(c) $t^2 \dfrac{d^2x}{dt^2} + t\dfrac{dx}{dt} + x = 0$ This is linear.

(d) $\dfrac{dr}{d\theta} + \theta r = 2$ This is also linear.

(2) Identify the independent variable, the dependent variable, and the order of each ODE.

(a) $\dfrac{d^4x}{dy^4} - xy = 1$ Independent y Dependent x Order 4

(b) $\ddot{x} + 2\dot{x} + 4x = \cos t$ Independent t (time) Dependent x Order 2

(c) $\cos\theta \dfrac{d^2y}{d\theta^2} = \dfrac{d^3y}{d\theta^3}$ Independent θ Dependent y Order 3

(d) $x\dfrac{dx}{dt} - t^2 = xt$ Independent t Dependent x Order 1

(3) Solve the initial value problem. (Implicit or explicit, your choice.)

$$dy + 2xy\,dx = 0 \qquad y(0) = 2$$

$dy = -2xy\,dx \quad \Rightarrow \quad \frac{1}{y}\,dy = -2x\,dx$

$\int \frac{1}{y}\,dy = -\int 2x\,dx \quad \Rightarrow \quad \ln|y| = -x^2 + C$

Applying $y(0)=2$ $\quad \ln 2 = C$

so implicitly $\quad \ln|y| = -x^2 + \ln 2$.

Explicitly $\quad y = 2e^{-x^2}$.

(4) Solve the first order separable ODE. (Implicit or explicit, your choice.)

$\frac{dy}{dx} = \frac{4x}{ye^y} \qquad ye^y \frac{dy}{dx} = 4x \quad \Rightarrow \quad ye^y\,dy = 4x\,dx$

$\int ye^y\,dy = \int 4x\,dx$

on the left

$u = y \quad du = dy$

$v = e^y \quad dv = e^y\,dy$

$ye^y - \int e^y = 2x^2 + C$

$ye^y - e^y = 2x^2 + C$

This is a one parameter family of solutions defined implicitly.

(5) Find all values of m such that $y = e^{mx}$ is a solution of the differential equation

$$y'' - 5y' + 4y = 0.$$

$$\left.\begin{aligned} y &= e^{mx} \\ y' &= me^{mx} \\ y'' &= m^2 e^{mx} \end{aligned}\right\} \overset{\text{substitute}}{\Longrightarrow} \quad y'' - 5y' + 4y = m^2 e^{mx} - 5me^{mx} + 4e^{mx} = 0$$

$$e^{mx}(m^2 - 5m + 4) = 0$$

This holds if $m^2 - 5m + 4 = 0$

$(m-1)(m-4) = 0$

$m = 1$ or $m = 4$

Hence $y = e^{mx}$ solves the ODE if

$m = 1$ or $m = 4$.

(6) Verify that for any constant c, the function $y = cx\ln x + 4$ is a solution of the ODE

$$x^2y'' - xy' + y = 4$$

$$\left.\begin{aligned} y &= cx\ln x + 4 \\ y' &= c\ln x + cx\cdot\tfrac{1}{x} = c\ln x + c \\ y'' &= \tfrac{c}{x} \end{aligned}\right\} \Longrightarrow \quad x^2y'' - xy' + y =$$

$$x^2\left(\frac{c}{x}\right) - x(c\ln x + c) + cx\ln x + 4 =$$

$$cx - cx\ln x - cx + cx\ln x + 4 =$$

$$4 = 4$$

an identity.

Hence $y = cx\ln x + 4$ solves the ODE for any c.

(7) Solve the initial value problem.

$$\frac{d^2y}{dx^2} = 9e^{3x} - 12x^2, \quad y(0) = 2, \quad y'(0) = 0$$

$y' = \int y''\,dx = \int (9e^{3x} - 12x^2)\,dx = 3e^{3x} - 4x^3 + c_1$

$y'(0) = 3e^0 - 0 + c_1 = 0 \Rightarrow c_1 = -3$

$y = \int y'\,dx = \int (3e^{3x} - 4x^3 - 3)\,dx = e^{3x} - x^4 - 3x + c_2$

$y(0) = e^0 - 0 - 0 + c_2 = 2 \Rightarrow c_2 = 1$

The solution to the IVP is

$y = e^{3x} - x^4 - 3x + 1$.

(8) Given that $y = c_1e^{2x} + c_2e^{-2x}$ is a two parameter family of solutions of $y'' - 4y = 0$, find the solution of the initial value problem

$$y'' - 4y = 0, \quad y(0) = -1, \quad y'(0) = 10$$

$y = c_1 e^{2x} + c_2 e^{-2x}$

$y' = 2c_1 e^{2x} - 2c_2 e^{-2x}$

$y(0) = c_1 + c_2 = -1$

$y'(0) = 2c_1 - 2c_2 = 10$

From the 1st eqn

$2c_1 + 2c_2 = -2$

$2c_1 - 2c_2 = 10$

add $4c_1 = 8 \Rightarrow c_1 = 2$

$c_2 = -1 - c_1 = -3$

The solution is

$y = 2e^{2x} - 3e^{-2x}$.