Exam 2 Math 2254 sec. 001

Summer 2015

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
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5	
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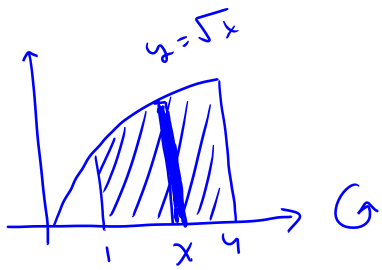
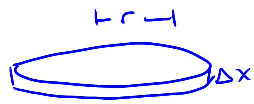
10	

INSTRUCTIONS: There are 10 problems worth 10 points each. No use of notes, books, or calculator is allowed. Illicit use of notes, a book, calculator, or any smart device will result in a grade of zero on this exam and may result in a formal allegation of academic misconduct. Show all of your work on the paper provided to receive full credit.

(1) The region bounded by the graph of $y = \sqrt{x}$, the x-axis, and the lines $x = 1$ and $x = 4$ is rotated about the x-axis. Find the volume of the resulting solid. (Use any applicable method.)





radius

$r = \sqrt{x}$

Volume of one disk

$= \pi r^2 \Delta x$

$= \pi (\sqrt{x})^2 \Delta x$

$= \pi x \,\Delta x$

The total Volume is

$$V = \int_1^4 \pi x \, dx = \frac{\pi x^2}{2} \Big|_1^4 = \pi \frac{4^2}{2} - \pi \frac{1^2}{2}$$

$$= \pi \left(8 - \frac{1}{2}\right) = \frac{15\pi}{2}$$

(2) Determine if each statement is *True* or *False*. Indicate your answer with T or F in the space provided.

(a) If using the shell method to find the volume of a solid revolved about the x-axis, the integration is with respect to x. F

(b) If f and g are continuous on $[a, b]$ and $f(x) \geq g(x) \geq 0$ on the interval. Then the volume V of the solid of revolution obtained by revolving the region bounded between f and g and between $x = a$ and $x = b$ about the x-axis is given by

$$V = \int_a^b \pi \left[(f(x))^2 - (g(x))^2\right] dx$$ T

(c) The arc length s of the continuous curve $y = f(x)$ from $x = a$ to $x = b$ is given by

$$s = \int_a^b \sqrt{1 + y^2}\, dx$$ F

(3) Evaluate the indefinite integral using any applicable method.

$$\int \ln(x^2)\, dx$$

Parts:

$u = \ln(x^2)$ $\quad du = \frac{2x}{x^2} dx = \frac{2}{x} dx$

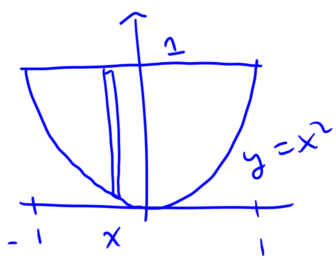
$v = x$ $\quad dv = dx$

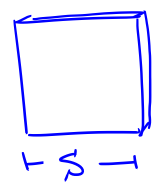
$$\int \ln(x^2) dx = x \ln(x^2) - \int x \cdot \frac{2}{x} dx$$

$$= x \ln(x^2) - \int 2\, dx$$

$$= x \ln(x^2) - 2x + C$$

(4) A solid has as its base the region bounded between the curves $y = x^2$ and $y = 1$. Cross sections taken perpendicular to the x-axis are squares with one side in the xy-plane. Find the volume of the solid. (The answer is $\frac{16}{15}$.)





$S = 1 - x^2$

so

$A(x) = s^2 = (1-x^2)^2$

$$V = \int_{-1}^{1} (1-x^2)^2\,dx$$

$$= \int_{-1}^{1} (1 - 2x^2 + x^4)\,dx = 2\int_{0}^{1} (1 - 2x^2 + x^4)\,dx$$

(by symmetry)

$$= 2\left[x - \frac{2}{3}x^3 + \frac{x^5}{5}\right|_0^1$$

$$= 2\left[1 - \frac{2}{3} + \frac{1}{5}\right] = 2\left(\frac{1}{3} + \frac{1}{5}\right) = 2\left(\frac{8}{15}\right) = \frac{16}{15}$$

(5) Set up, **but DO NOT evaluate**, an integral for the arc length of the curve $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$.

$$\frac{dy}{dx} = \cos x \quad , \qquad 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \cos^2 x$$

$$S = \int_0^{\pi/2} \sqrt{1 + \cos^2 x} \; dx$$

(6) A 50 N force is required to stretch a spring 10 cm (0.1 m) from its natural length.

(a) Find the spring constant k. $\qquad F = kx$

(b) Find the work done stretching the srping 10 cm from its natural length.

a) $50\,N = (0.1\,m)k \Rightarrow k = \frac{50\,N}{0.1\,m} = 500\ \frac{N}{m}$

b)
$$W = \int_0^{0.1} 500\ \tfrac{N}{m}\ x\,m \quad dx\,m$$

$$= \int_0^{0.1} 500x\ dx \qquad Nm$$

$$= 500\ \frac{x^2}{2}\Big|_0^{0.1} \qquad J$$

$$= 250\ (0.1)^2 \quad J$$

$$= 250\ (0.01)\ J \quad = 2.5 \quad J$$

(7) Evaluate the indefinite integral using any applicable method.

$$\int \sin^3 x\, dx = \int \sin^2 x \sin x\, dx$$

$$= \int (1-\cos^2 x) \sin x\, dx$$

$u = \cos x$

$du = -\sin x\, dx$

$$= -\int (1-u^2)\, du$$

$$= -u + \frac{u^3}{3} + C$$

$$= -\cos x + \frac{\cos^3 x}{3} + C$$

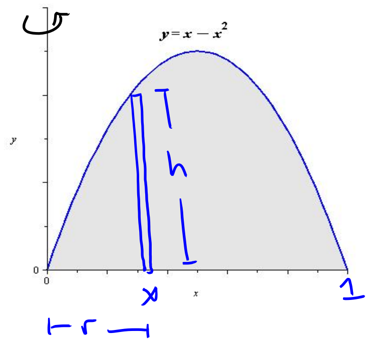
(8) Evaluate the indefinite integral using any applicable method.

$$\int \cos^2 x\, dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$$

$$= \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{2}\sin 2x + C$$

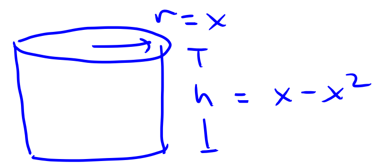
$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

(9) The region bounded by the curve $y = x - x^2$ and the x-axis is rotated about the y-axis to generate a solid. Find the volume of the solid.



Shells are called for:

$x - x^2 = 0 \Rightarrow x(x-1) = 0 \quad x = 0 \text{ or } x = 1$



Volume of shell $= 2\pi r h \Delta x$

$= 2\pi x (x - x^2) \Delta x$

The total volume is

$$V = \int_0^1 2\pi x\,(x - x^2)\,dx$$

$$= 2\pi \int_0^1 (x^2 - x^3)\,dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4}\right|_0^1 = 2\pi\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{2\pi}{12} = \frac{\pi}{6}$$

(10) A variable force of $F(x) = 2^x$ N is required to move an object along the x-axis from $x = 1$ m to $x = 2$m. Find the work done.

$$W = \int_1^2 F(x)\text{ N } dx \text{ m} = \int_1^2 2^x\,dx \quad \text{Nm}$$

$$= \frac{1}{\ln 2}\, 2^x \Big|_1^2 \text{ Nm}$$

$$= \frac{1}{\ln 2}\left[2^2 - 2^1\right] \text{ J}$$

$$= \frac{2}{\ln 2} \text{ J}$$