Exam 2 Math 2306 sec. 54

Fall 2015

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

Problem	Points
1	
2	
3	
4	
5	

INSTRUCTIONS: There are 5 problems worth 20 points each. You may use one sheet (8.5" $\times$ 11") of your own prepared notes/formulas.

No use of a calculator, text book, smart device, or other resource is permitted. Illicit use of any additional resource will result in a grade of zero on this exam as well as a formal allegation of academic misconduct. Show all of your work on the paper provided to receive full credit.

(1) For each set of functions, determine whether they are linearly dependent or independent on the indicated interval. (Clearly state your conclusion with justification.)

(a) $y_1 = 2x^3, \quad y_2 = 6x, \quad y_3 = 2x - 4x^3, \quad (0, \infty)$

Note $2y_1 + y_3 - \frac{1}{3}y_2 = 4x^3 + 2x - 4x^3 - \frac{1}{3}(6x)$

$= 4x^3 + 2x - 4x^3 - 2x = 0$ for all x in $(0, \infty)$

So for $c_1 = 2$, $c_2 = \frac{-1}{3}$, $c_3 = 1$ (not all zeros)

$c_1 y_1 + c_2 y_2 + c_3 y_3 = 0$ for all x in I.

Hence they are linearly dependent.

(b) $f_1(x) = e^{3x}, \quad f_2(x) = 2xe^{3x}, \quad (-\infty, \infty)$

$$W(f_1, f_2)(x) = \begin{vmatrix} e^{3x} & 2x e^{3x} \\ 3e^{3x} & 2e^{3x} + 6xe^{3x} \end{vmatrix}$$

$= 2e^{6x} + 6xe^{6x} - 6xe^{6x} = 2e^{6x} \neq 0$

$W \neq 0$; they are linearly Independent.

(2) Find the general solution of each differential equation.

(a) $y'' - 14y' + 49y = 0$

$m^2 - 14m + 49 = 0 \qquad (m-7)^2 = 0$

$m = 7$ repeated root

$y_1 = e^{7x}, \quad y_2 = xe^{7x}$

$$\boxed{y = C_1 e^{7x} + C_2 x e^{7x}}$$

(b) $y'' - 4y' + 13y = 0$

$m^2 - 4m + 13 = 0$

$m^2 - 4m + 4 + 9 = 0 \qquad (m-2)^2 = -9$

$m - 2 = \pm 3i$

$m = 2 \pm 3i$

$y_1 = e^{2x} \cos(3x), \quad y_2 = e^{2x} \sin(3x)$

$\alpha = 2, \ \beta = 3$

$$\boxed{y = C_1 e^{2x} \cos(3x) + C_2 e^{2x} \sin(3x)}$$

(3) For each nonhomogeneous DE, determine the *form* of the particular solution when using the method of undetermined coefficients. **Do not solve for any coefficients A, B, etc.** (You may wish to refer to results of problem (2).)

(a) $y'' - 14y' + 49y = 3e^{7x} - x^3$ $\qquad y_c = C_1 e^{7x} + C_2 x e^{7x}$ $\quad$ (2a)

$y_{p_1} = A e^{7x}$ ← solves the homogeneous eqn.

Corrected $\quad y_{p_1} = A x^2 e^{7x}$

$y_{p_2} = Bx^3 + Cx^2 + Dx + E$

$$\boxed{y_p = A x^2 e^{7x} + Bx^3 + Cx^2 + Dx + E}$$

(b) $y'' - 4y' + 13y = 2x\sin(2x)$ $\qquad y_c = C_1 e^{2x} \cos 3x + C_2 e^{2x} \sin 3x$ $\quad$ (2b)

$$\boxed{y_p = (Ax+B)\sin(2x) + (Cx+D)\cos(2x)}$$

No modification is required.

(c) $y'' - 4y' + 13y = e^{2x}\sin(3x)$ $\qquad$ Same y_c as above

$y_p = A e^{2x} \sin(3x) + B e^{2x} \cos(3x)$ → this is y_c.

Corrected
$$\boxed{y_p = A x e^{2x} \sin(3x) + B x e^{2x} \cos(3x)}$$

(4) Find the general solution of the nonhomogeneous differential equation.

$$y'' + 3y' - 4y = -12x$$

Get y_c: $m^2 + 3m - 4 = 0$ $(m+4)(m-1) = 0$

$m_1 = -4$, $m_2 = 1$

$y_c = C_1 e^{-4x} + C_2 e^{x}$

Get y_p: Assume $y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$y_p'' + 3y_p' - 4y_p = -12x$

$0 + 3A - 4(Ax + B) = -12x$

$-4 \cdot Ax + (3A - 4B) = -12x$

$-4A = -12 \Rightarrow A = 3$

$3A - 4B = 0 \Rightarrow B = \frac{3}{4} A = \frac{9}{4}$

$y_p = 3x + \frac{9}{4}$

$$\boxed{y = C_1 e^{-4x} + C_2 e^{x} + 3x + \frac{9}{4}}$$

(5) Consider the homogeneous differential equation for which one solution is given.

$$4x^2y'' + y = 0; \qquad y_1 = x^{1/2}$$

(a) Find a second linearly independent solution y_2. $y'' + \frac{1}{4x^2}y = 0 \qquad P(x) = 0$

$u = \int \frac{e^{-\int P(x)dx}}{y_1^2} dx \qquad e^{-\int P(x)dx} = e^0 = 1$

$= \int \frac{1}{(x^{1/2})^2} dx = \int \frac{1}{x} dx$

$= \ln x \qquad y_2 = y_1 u = x^{1/2} \ln x$

$$\boxed{y_2 = x^{1/2} \ln x}$$

(b) Solve the initial value problem $4x^2y'' + y = 0$, $y(1) = 4$, $y'(1) = 1$.

$y = C_1 x^{1/2} + C_2 x^{1/2} \ln x$

$y' = \frac{1}{2} C_1 x^{-1/2} + \frac{1}{2} C_2 x^{-1/2} \ln x + C_2 \frac{x^{1/2}}{x}$

$y(1) = C_1 (1) + C_2 \ln(1) = C_1 = 4 \quad \Rightarrow C_1 = 4$

$y'(1) = \frac{1}{2} C_1 + \frac{1}{2} C_2 \ln 1 + C_2 = 1$

$C_2 = 1 - \frac{1}{2} C_1 = 1 - 2 = -1$

$$\boxed{y = 4x^{1/2} - x^{1/2} \ln x}$$