Exam 3 Math 2306 sec. 58

Spring 2016

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	

INSTRUCTIONS: There are 5 problems worth 20 points each. You may use one sheet (8.5" × 11") of your own prepared notes/formulas.

No use of a calculator, text book, smart device, or other resource is permitted. Illicit use of any additional resource will result in a grade of zero on this exam as well as a formal allegation of academic misconduct. Show all of your work on the paper provided to receive full credit.

(1) For each homogeneous equation, either write out the characteristic equation or if one does not exists briefly state why.

(a) $x^2y''+2xy'-3y=0$

None exists. The equation is not constant coefficient.

(b) $y^{(4)}+3y'''-2y''+2y=0$

$m^4+3m^3-2m^2+2=0$

(c) $\dfrac{d^6y}{dx^6}+4y=0$

$m^6+4=0$

(d) $3y''+2yy'+y=0$

None exists. The equation is not linear due to the yy' term.

(2) Find the general solution of each homogeneous equation.

(a) $y''-2y'-8y=0$

$m^2-2m-8=0$

$(m-4)(m+2)=0 \implies m=4$ or $m=-2$

$y=c_1e^{4x}+c_2e^{-2x}$

(b) $y''-6y'+10y=0$

$m^2-6m+10=0$

$m^2-6m+9+1=0$

$(m-3)^2=-1$

$m=3\pm i$

$\alpha=3,\ \beta=1$

$y=c_1e^{3x}\cos x+c_2e^{3x}\sin x$

(c) $y''+8y'+16y=0$

$m^2+8m+16=0 \quad (m+4)^2=0 \implies m=-4$ repeated

$y=c_1e^{-4x}+c_2xe^{-4x}$

(d) $y''-4y=0$

$m^2-4=0 \implies m=\pm 2$

$y=c_1e^{2x}+c_2e^{-2x}$

(3) For each nonhomogeneous equation, determine the **form** of the particular solution when using the method of undetermined coefficients. DO NOT bother trying to find any of the coefficients A, B, etc.

(a) $y''-2y'-8y = xe^{-2x}+1$

$y_c = c_1 e^{4x} + c_2 e^{-2x}$ 2(a)

$g_1(x) = xe^{-2x}$, $y_{p_1} = (Ax+B)e^{-2x}$ This won't work

$y_{p_1} = (Ax+B)xe^{-2x}$

$g_2(x) = 1$ $y_{p_2} = C$

$$\boxed{y_p = (Ax^2+Bx)e^{-2x} + C}$$

(b) $y''-6y'+10y = e^{3x}\cos x$

$y_c = c_1 e^{3x}\cos x + c_2 e^{3x}\sin x$ 2(b)

$y_p = \left(Ae^{3x}\cos x + Be^{3x}\sin x\right)x$

$$\boxed{y_p = Axe^{3x}\cos x + Bxe^{3x}\sin x}$$

(c) $y''+8y'+16y = 4x^2+16e^{-4x}$

$y_c = c_1 e^{-4x} + c_2 xe^{-4x}$ 2(c)

$g_1(x) = 4x^2$ $y_{p_1} = Ax^2+Bx+C$

$g_2(x) = 16e^{-4x}$ $y_{p_2} = De^{-4x}$ this won't work

$y_{p_2} = Dx^2e^{-4x}$

$$\boxed{y_p = Ax^2+Bx+C+Dx^2e^{-4x}}$$

(4) Solve the initial value problem.

$$y'' + y' = 12e^{2x}, \quad y(0) = 3, \quad y'(0) = 5$$

Find y_c: $m^2 + m = 0 \Rightarrow m(m+1) = 0$ $m = 0$ or $m = -1$

$y_c = C_1 + C_2 e^{-x}$

For y_p: Try $y_p = Ae^{2x}$ $y_p'' + y_p' = 12e^{2x}$

$y_p' = 2Ae^{2x}$ $4Ae^{2x} + 2Ae^{2x} = 12e^{2x}$

$y_p'' = 4Ae^{2x}$ $6A = 12 \Rightarrow A = 2$

So $y_p = 2e^{2x}$

Hence $y = C_1 + C_2 e^{-x} + 2e^{2x}$.

$y' = -C_2 e^{-x} + 4e^{2x}$

$y(0) = C_1 + C_2 + 2 = 3$ $\Rightarrow$ $C_1 + C_2 = 1$

$y'(0) = -C_2 + 4 = 5$ $\quad -C_2 = 1 \Rightarrow C_2 = -1$

$C_1 - 1 = 1 \Rightarrow C_1 = 2$

$$\boxed{\text{Finally, } y = 2 - e^{-x} + 2e^{2x}.}$$

(5) Find the general solution of the nonhomogeneous equation.

$$y'' - 3y' + 2y = 2x^2$$

Find y_c : $m^2 - 3m + 2 = 0$ $(m-2)(m-1) = 0$

$m = 2$ or $m = 1$

$$y_c = C_1 e^{2x} + C_2 e^{x}$$

For y_p try $y_p = Ax^2 + Bx + C$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

$$y_p'' - 3y_p' + 2y_p = 2x^2$$

$$2A - 3(2Ax + B) + 2(Ax^2 + Bx + C) = 2x^2$$

$$2Ax^2 + (-6A + 2B)x + (2A - 3B + 2C) = 2x^2$$

$2A = 2 \Rightarrow A = 1$

$-6A + 2B = 0 \Rightarrow B = 3A = 3$

$2A - 3B + 2C = 0 \Rightarrow 2C = 3B - 2A = 9 - 2 = 7$

$C = 7/2$

$$\boxed{y = C_1 e^{2x} + C_2 e^{x} + x^2 + 3x + 7/2}$$