Exam 3 Math 2306 sec. 60 Spring 2019

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	

INSTRUCTIONS: There are 5 problems worth 20 points each. You may use one sheet (8.5" $\times$ 11") of your own prepared notes/formulas.
No use of a calculator, text book, smart device, or other resource is permitted. Illicit use of any additional resource will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.
Show all of your work on the paper provided to receive full credit.

1. Find the general solution of each homogeneous equation.

(a) $y'' - 49y = 0$

$m^2 - 49 = 0$

$m^2 = 49$

$m = \pm 7$

$y = c_1 e^{7x} + c_2 e^{-7x}$

(b) $y'' - 2y' + 5y = 0$

$m^2 - 2m + 5 = 0$

$m^2 - 2m + 1 + 4 = 0$

$(m-1)^2 = -4$

$m - 1 = \pm 2i$

$m = 1 \pm 2i$

$y = C_1 e^{x} \cos(2x) + C_2 e^{x} \sin(2x)$

2. A dying battery imparts an electromotive force $E(t) = 40e^{-t}$ to an LC-series circuit (there is no resistor). The inductance is 4 henry and the capacitance is 0.0025 (i.e. $\frac{1}{400}$) farads. If the initial charge on the capacitor $q(0) = 0$ and the initial current $i(0) = 0$, determine the charge $q(t)$ for all $t > 0$.

$L = 4$, $C = \frac{1}{400}$, $E = 40e^{-t}$ and $R = 0$

$Lq'' + Rq' + \frac{1}{C}q = E \Rightarrow 4q'' + 400q = 40e^{-t}$

In standard form $q'' + 100q = 10e^{-t}$

For q_c: $m^2 + 100 = 0$ $m = \pm 10i$

$q_c = C_1 \cos(10t) + C_2 \sin(10t)$

For q_p, set $q_p = Ae^{-t}$ $q_p' = -Ae^{-t}$, $q_p'' = Ae^{-t}$

$Ae^{-t} + 100Ae^{-t} = 10e^{-t} \Rightarrow 101A = 10$ $A = \frac{10}{101}$

Then $q = C_1 \cos(10t) + C_2 \sin(10t) + \frac{10}{101}e^{-t}$

$q' = -10C_1 \sin(10t) + 10C_2 \cos(10t) - \frac{10}{101}e^{-t}$

$q(0) = C_1 + \frac{10}{101} \Rightarrow C_1 = \frac{-10}{101}$

$q'(0) = 10C_2 - \frac{10}{101} \Rightarrow C_2 = \frac{1}{10}\left(\frac{10}{101}\right) = \frac{1}{101}$

The Charge

$$q(t) = \frac{-10}{101}\cos(10t) + \frac{1}{101}\sin(10t) + \frac{10}{101}e^{-t}$$

for $t > 0$

3. Find the general solution of the nonhomogenous equation.

$$y'' - 2y' + y = \frac{6e^t}{\sqrt{t}}$$

For y_c: $m^2 - 2m + 1 = 0$ $(m-1)^2 = 0$

$m = 1$ repeated

$y_1 = e^t$, $y_2 = te^t$

Using variation of parameters, $y_p = u_1 y_1 + u_2 y_2$

~~Here~~ $g(t) = \frac{6e^t}{\sqrt{t}}$ and $W = \begin{vmatrix} e^t & te^t \\ e^t & e^t + te^t \end{vmatrix} = e^{2t} + te^{2t} - te^{2t}$

$= e^{2t}$

$u_1 = \int \frac{-g y_2}{W} dt = \int \frac{\frac{-6}{\sqrt{t}} e^t (te^t)}{e^{2t}} dt = -6 \int \sqrt{t}\, dt = -6 \frac{t^{3/2}}{3/2} = -4t^{3/2}$

$u_2 = \int \frac{g y_1}{W} dt = \int \frac{\frac{6}{\sqrt{t}} e^t (e^t)}{e^{2t}} dt = 6 \int t^{-1/2} dt = 6 \frac{t^{1/2}}{1/2} = 12\sqrt{t}$

$y_p = -4t^{3/2} e^t + 12 t^{1/2} t e^t = 8 t^{3/2} e^t$

The solution

$y = c_1 e^t + c_2 t e^t + 8 t^{3/2} e^t$

4. Suppose you wish to use the method of undetermined coefficients to find a particular solution to the differential equation

$$y'' - 7y' + 10y = g(x).$$

For each function $g(x)$, determine the form of the particular solution. (Do not solve for any of the coefficients A, B, etc.)

Consider y_c

$m^2 - 7m + 10 = 0$

$(m-5)(m-2) = 0$

$m = 5$ or $m = 2$

$y_c = C_1 e^{5x} + C_2 e^{2x}$

(a) $g(x) = 4xe^{5x} - 2\cos(2x)$

S_1 S_2

$y_{p_1} = (Ax+B)e^{5x} \cdot x$

$= (Ax^2 + Bx)e^{5x}$

$y_{p_2} = C\cos(2x) + D\sin(2x)$

$y_p = (Ax^2 + Bx)e^{5x} + C\cos(2x) + D\sin(2x)$

(b) $g(x) = 4xe^x \cos(\pi x)$

$y_p = (Ax+B)e^x \cos(\pi x) + (Cx+D)e^x \sin(\pi x)$

5. Evaluate each Laplace transform or inverse Laplace transform.

(a) $\mathscr{L}\{3\sin(2t) - 2t^4\}$ $= 3\mathscr{L}\{\sin(2t)\} - 2\mathscr{L}\{t^4\}$

$$= 3\,\frac{2}{s^2+4} - 2\,\frac{4!}{s^5} = \frac{6}{s^2+4} - \frac{48}{s^5}$$

(b) $\mathscr{L}\{(2+e^{3t})^2\}$ $= \mathscr{L}\{4 + 4e^{3t} + e^{6t}\} = 4\mathscr{L}\{1\} + 4\mathscr{L}\{e^{3t}\} + \mathscr{L}\{e^{6t}\}$

$$= \frac{4}{s} + \frac{4}{s-3} + \frac{1}{s-6}$$

(c) $\mathscr{L}^{-1}\left\{\frac{1}{s^3} - \frac{1}{s^2+16}\right\}$ $= \mathscr{L}^{-1}\left\{\frac{1}{2!}\,\frac{2!}{s^3}\right\} - \mathscr{L}^{-1}\left\{\frac{1}{4}\,\frac{4}{s^2+16}\right\}$

$$= \frac{1}{2}t^2 - \frac{1}{4}\sin(4t)$$

(d) $\mathscr{L}^{-1}\left\{\frac{5s}{(s-1)(s+4)}\right\}$ $= \mathscr{L}^{-1}\left\{\frac{1}{s-1}\right\} + 4\mathscr{L}^{-1}\left\{\frac{1}{s+4}\right\}$

$$= e^{t} + 4e^{-4t}$$

$$\frac{5s}{(s-1)(s+4)} = \frac{A}{s-1} + \frac{B}{s+4} \Rightarrow 5s = A(s+4) + B(s-1)$$

$s=1$ $\quad 5 = 5A \quad A=1$

$s=-4$ $\quad -20 = -5B \quad B=4$