## Exam 4 Math 2306 sec. 54

Fall 2015

Name: $\qquad$
Your signature (required) confirms that you agree to practice academic honesty.

## Signature:

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| Problem | Points |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

INSTRUCTIONS: There are 5 problems worth 20 points each. You may use one sheet ( 8.5 " $\times 11$ ") of your own prepared notes/formulas and the provided table of Laplace transforms.

No use of a calculator, text book, smart device, or other resource is permitted. Illicit use of any additional resource will result in a grade of zero on this exam as well as a formal allegation of academic misconduct. Show all of your work on the paper provided to receive full credit.
(1) (a) Find the half range sine series of $f$. Then, (b) on the graph provided, plot three periods over the interval $(-3 p, 3 p)$ of the sine series found in part (a).

$$
f(x)=1, \quad 0<x<\pi
$$

$$
\left.\begin{array}{rl}
f(x) & =\sum_{n=1}^{\infty} b_{n} \sin (n x) \text { where } b_{n}
\end{array}=\frac{2}{\pi} \int_{0}^{\pi} f(x) \sin (n x) d x\right] \text { bn } \begin{aligned}
b_{n} & =\frac{2}{\pi} \int_{0}^{\pi} 1 \cdot \sin (n x) d x=\left.\frac{-2}{n \pi} \cos (n x)\right|_{0} ^{\pi} \\
& =\frac{-2}{n \pi}(\cos (n \pi)-\cos 0)
\end{aligned}
$$

$$
\text { a) } f(x)=\sum_{n=1}^{\infty} \frac{2}{n \pi}\left(1-(-1)^{n}\right) \sin (n x)
$$


(2) Use any method to evaluate the Laplace transform or inverse transform as indicated.
(a) $\mathscr{L}\left\{(t-4)^{3} \mathscr{U}(t-4)\right\}=e^{-4 s} \mathscr{L}\left\{t^{3}\right\}=\frac{3!e^{-4 s}}{s^{4}}$
(b) $\mathscr{L}^{-1}\left\{\frac{e^{-\pi s}}{s^{2}}\right\}=(t-\pi) u(t-\pi)$
(c) $\mathscr{L}^{-1}\left\{\frac{s}{(s+3)^{2}+9}\right\}=\mathscr{L}^{-1}\left\{\frac{s+3-3}{(s+3)^{2}+9}\right\}=$

$$
=\mathcal{L}^{-1}\left\{\frac{s+3}{(s+3)^{2}+9}\right\}-\mathcal{L}^{-1}\left\{\frac{3}{(s+3)^{2}+9}\right\}
$$

$$
=e^{-3 t} \cos 3 t-e^{-3 t} \sin 3 t
$$

(d)

$$
\mathscr{L}\left\{e^{-0.1 t} \sin (2 t)\right\}=\frac{2}{(5+0.1)^{2}+4}
$$

(3) Solve the initial value problem using the method of Laplace transforms.

$$
\begin{aligned}
& y^{\prime}+3 y=3 t^{2} e^{-3 t}, \quad y(0)=2 \\
& \text { Lat } \mathscr{L}\{y\}=\Psi(s) \\
& \mathcal{L}\left\{y^{\prime}+3 y\right\}=\mathcal{L}\left\{3 t^{2} e^{-3 t}\right\} \\
& s Y(s)-y(0)+3 Y(\sigma)=3 \frac{2!}{(s+3)^{3}} \\
& (s+3) \varphi(6)=\frac{6}{(s+3)^{3}}+2 \\
& Y(s)=\frac{6}{(s+3)^{4}}+\frac{2}{s+3}=\frac{3!}{(s+3)^{4}}+\frac{2}{s+3} \\
& y(t)=\mathcal{L}^{-1}\{Y(s)\} \\
& =t^{3} e^{-3 t}+2 e^{-3 t}
\end{aligned}
$$

(4) Find the Fourier series of $f$.

$$
\begin{aligned}
f(x)= & \begin{array}{ll}
1, \quad-1<x<0 \\
3, & 0 \leq x<1
\end{array} \\
a_{0} & =\frac{1}{1} \int_{-1}^{1} f(x) d x=\int_{-1}^{0} d x+\int_{0}^{1} 3 d x=\left.x\right|_{-1} ^{0}+\left.3 x\right|_{0} ^{1}=1+3=4 \\
a_{n} & =\frac{1}{1} \int_{-1}^{1} f(x) \cos (n \pi x) d x=\int_{-1}^{0} \cos (n \pi x) d x+\int_{0}^{1} 3 \cos (n \pi x) d x \\
& =\left.\frac{1}{n \pi} \sin (n \pi x)\right|_{-1} ^{0}+\left.\frac{3}{n \pi} \sin (n \pi x)\right|_{0} ^{1}=0 \\
b_{n} & =\frac{1}{1} \int_{-1}^{1} f(x) \sin (n \pi x) d x=\int_{-1}^{0} \sin (n \pi x) d x+\int_{0}^{1} 3 \sin (n \pi x) d x \\
& =\left.\frac{-1}{n \pi} \cos (n \pi x)\right|_{-1} ^{1}+\left.\frac{-3}{n \pi} \cos (n \pi x)\right|_{0} ^{1} \\
& =\frac{-1}{n \pi}[\cos 0-\cos (-n \pi)]-\frac{3}{n \pi}[\cos (n \pi)-\cos 0] \\
& =\frac{-1}{n \pi}+\frac{1}{n \pi}(-1)^{n}-\frac{3}{n \pi}(-1)^{n}+\frac{3}{n \pi}=\frac{2}{n \pi}\left(1-(-1)^{n}\right) \\
& =2+\sum_{n=1}^{\infty} \frac{2}{n \pi}\left(1-(-1)^{n}\right) \sin (n \pi x)
\end{aligned}
$$

(5) An LR series circuit with inductance 1 h and resistance 6 ohms is attached to a battery with an open switch. After 1 second, the switch is closed applying a constant 60 volts. Use the method of Laplace transforms to find the current if the initial current is $i(0)=0 \mathrm{~A}$. That is, solve the IVP

$$
\begin{aligned}
& \frac{d i}{d t}+6 i=\left\{\begin{array}{lc}
0, & 0 \leq t<1 \\
60, & t \geq 1
\end{array} \quad i(0)=0\right. \\
& =60 u(t-1) \\
& y\left\{i^{\prime}+6 i\right\}=\mathcal{L}\{6 \mathrm{bu}(t-1)\} \\
& s I(s)-i(0)+6 I(s)=\frac{60}{s} e^{-s} \\
& (s+6) I(s)=\frac{60 e^{-5}}{s} \\
& I(s)=\frac{60 e^{-s}}{s(s+6)} \\
& \text { Decompose : } \frac{60}{s(s+6)}=\frac{A}{5}+\frac{B}{s+6} \Rightarrow 60=A(s+6)+B s \\
& \text { Let } s=0 \quad 60=6 A \Rightarrow A=10 \\
& S=-6 \quad 60=-6 \beta \Rightarrow \beta=-10 \\
& I(s)=\frac{10}{s} e^{-s}-\frac{10}{s+6} e^{-s} \\
& \hat{i}(t)=\mathcal{L}^{-1}\{I(s)\} \\
& =10 u(t-1)-10 e^{-6(t-1)} u(t-1)
\end{aligned}
$$

