Look Back Problems

What is the general format of a

(a) first order linear ODE? $\frac{dy}{dx} + P(x)y = f(x)$

use integrating factor $\mu = e^{\int P(x)dx}$

(b) first order separable ODE? $\frac{dy}{dx} = g(x)h(y)$

$$\int \frac{1}{h(y)}dy = \int g(x)dx$$

(c) Bernoulli equation?

$$\frac{dy}{dx} + P(x)y = f(x)y^n$$

$n \neq 0, n \neq 1$ Let $u = y^{1-n}$

Look Back Problems

$$x\frac{dy}{dx} = 2e^{x^2} - 2y$$

$$\frac{dy}{dx} = \frac{2e^{x^2} - 2y}{x}$$

Solve the IVP

$$\text{(a)} \quad x\frac{dy}{dx} + 2y = 2e^{x^2}, \quad y(1) = 0$$

Standard form $\quad \frac{dy}{dx} + \frac{2}{x}y = \frac{2e^{x^2}}{x}$

$$\text{(b)} \quad \frac{dy}{dx} = \frac{y^2+1}{x^2+1}, \quad y(0) = 1$$

separable

(a) Solution is $\quad y = \frac{e^{x^2} - e}{x^2}$

(b) Solution is $\quad \tan^{-1} y = \tan^{-1} x + \frac{\pi}{4}$

Look Back Problems

Solve the initial value problem. One solution to the ODE is given.

$$x^2y'' - 9xy' + 25y = 0, \quad y(1) = 1, \quad y'(1) = 0, \quad \text{one solution } y_1 = x^5$$

Use reduction of order

$$y_2 = y_1 \int \frac{e^{-\int P\,dx}}{(y_1)^2}\,dx$$

the solution is $y = x^5 - 5x^5 \ln x$

Find a particular solution using the method of undetermined coefficients.

$$y'' - 2y' - 3y = 3x^2$$

The particular solution to the 2^{nd} problem is $y_p = -x^2 + \frac{4}{3}x - \frac{14}{9}$

Look Back Problems

Determine whether the equation is exact. Solve the equation using a special integrating factor if needed.

It is exact.

$$(2x - 2y + y^3e^x)\,dx + (3y^2e^x - 2x)\,dy = 0$$

Solutions are defined by

$F(x,y) = C \Rightarrow x^2 - 2xy + y^3e^x = C$

An undamped spring mass system consists of a 5 kg mass attached to a spring with srping constant 24 N/m. An external driving force $f(t) = f_0\cos(\gamma t)$ is applied. Determine the differential equation governing the displacement $x(t)$ for $t > 0$. What is the frequency γ at which pure resonance will occur?

$\gamma = \omega$

and $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{24}{5}}$

resonance frequency $\gamma = \sqrt{\frac{24}{5}}$

Look Back Problems

A property of Laplace transforms is the following:

If $\mathscr{L}\{f(t)\} = F(s)$, then $\mathscr{L}\{t^n f(t)\} = (-1)^n \dfrac{d^n}{ds^n} F(s)$.

(a) Evaluate $\mathscr{L}\{t\sin(2t)\}$ $= -\frac{d}{ds}\frac{2}{s^2+4} = \frac{4s}{(s^2+4)^2}$

(b) Use this new result to evaluate $\mathscr{L}\{t^2e^t\}$, and compare this to the other method for evaluating $\mathscr{L}\{t^2e^t\}$.

$\frac{d^2}{ds^2}\frac{1}{(s-1)} = \frac{2}{(s-1)^3}$

also $\mathscr{L}\{t^2\} = \frac{2!}{s^3}$

so $\mathscr{L}\{t^2e^t\} = \frac{2!}{(s-1)^3}$