February 11 MATH 1112 sec. 54 Spring 2019

Summary of Log Properties

Assume each expression is well defined.

(i) Change of base: $\log_b(M) = \dfrac{\log_a(M)}{\log_a(b)}$

(ii) Log of Product: $\log_a(MN) = \log_a(M) + \log_a(N)$

(iii) Log of Power: $\log_a(M^p) = p\log_a(M)$

(iv) Log of Quotient: $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$

(v) Inverse Function: $a^{\log_a(x)} = x$ and $\log_a(a^x) = x$

(vi) Special Values: $\log_a(1) = 0$, $\log_a(a) = 1$, and $\log_a(0)$ is never defined.

Section 5.5: Solving Exponential and Logarithmic Equations

We can solve equations involving logs and exponentials by making use of the following.

Base-Exponent Equality For any $a > 0$ with $a \neq 1$, and for any real numbers x and y

$$a^x = a^y \quad \text{if and only if} \quad x = y.$$

Solving Exponential and Logarithmic Equations

Logarithm Equality For and $a > 0$ with $a \neq 1$, and for any positive numbers x and y

$$\log_a x = \log_a y \quad \text{if and only if} \quad x = y.$$

Solving Exponential and Logarithmic Equations

Inverse Function For any $a > 0$ with $a \neq 1$

$$a^{\log_a x} = x \quad \text{for every} \quad x > 0$$

$$\log_a(a^x) = x \quad \text{for every real} \quad x.$$

Example

Solve the equation $3^{2x+1} = 81$. Show that the same result is obtained by two approaches
(a) by using the fact that $81 = 3^4$ and equating exponents,

$3^{2x+1} = 81$ use $81 = 3^4$

$= 3^4$ use $a^x = a^y$ means $x = y$

$2x+1 = 4$

$2x = 3$

$x = \frac{3}{2}$

quick check:

$3^{2\left(\frac{3}{2}\right)+1} = 3^{3+1} = 3^4 = 81$ ✓

Example

Solve the equation $3^{2x+1} = 81$. Show that the same result is obtained by two approaches
(b) by using the base 3 logarithm as an inverse function.

$3^{2x+1} = 81$

Take log base 3

$\log_3\left(3^{2x+1}\right) = \log_3(81)$

$\log_a\left(a^z\right) = z$

$3^4 = 81$ so $\log_3(81) = 4$

$2x+1 = 4$

$2x = 3$

$x = \frac{3}{2}$

Example

Find an exact solution[1] to the equation

$2^{x+1} = 5^x$

Let's use the natural log. We'll get the x's out of the exponents.

$\ln(2^{x+1}) = \ln(5^x)$ Take a log of both sides

$(x+1)\ln 2 = x\ln 5$ $\log_a(M^r) = r\log_a M$

$x\ln 2 + 1\ln 2 = x\ln 5$ Solve for x

[1] An exact solution may be a number such as $\sqrt{2}$ or $\ln(7)$ which requires a calculator to approximate as a decimal.

$$\ln 2 = x\ln 5 - x\ln 2$$

$$\ln 2 = x(\ln 5 - \ln 2)$$

$$x = \frac{\ln 2}{\ln 5 - \ln 2}$$

Graphical Solution to $2^{x+1} = 5^x$

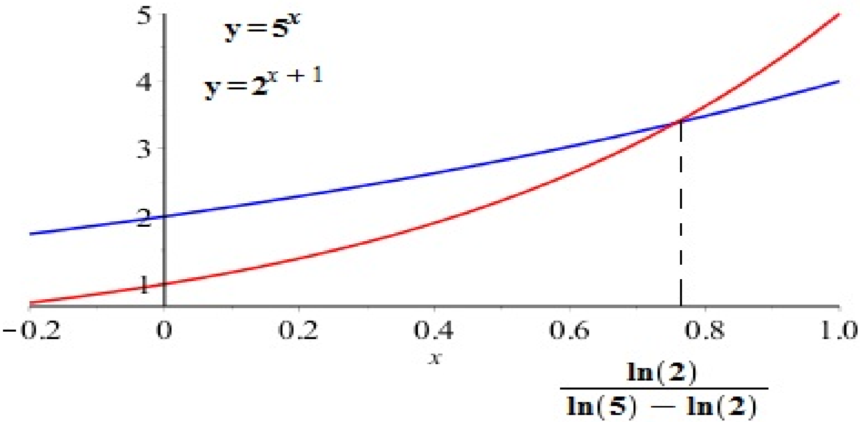

Figure: Plots of $y = 2^{x+1}$ and $y = 5^x$ together. The curves intersect at the solution $x = \ln 2/(\ln 5 - \ln 2) \approx 0.7565$.

Which curve is $y = 2^{x+1}$, red or blue?

Question

An exact solution to $3^{-x} = 4^{x-1}$ can be found using the natural logarithm. An exact solution is

(a) $x = \dfrac{\ln 4}{\ln 4 - \ln 3}$

(b) $x = \dfrac{\ln 3}{\ln 4 - \ln 3}$

(c) $x = \dfrac{\ln 4}{\ln 4 + \ln 3}$

(d) $x = \dfrac{\ln 3}{\ln 4 + \ln 3}$

(e) I know how to do this, but my answer is not here