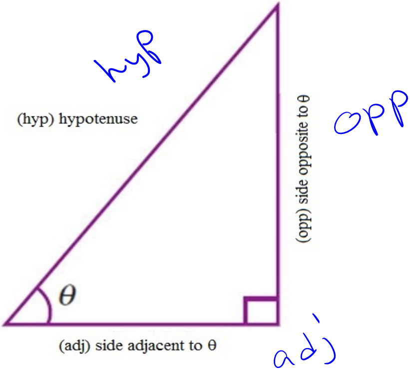
February 12 MATH 1112 sec. 54 Spring 2020

Trigonometric Functions of Acute Angles

In this section, we are going to define six new functions called **trigonometric functions.** We begin with an acute angle θ in a right triangle with the sides whose lengths are labeled:



Sine, Cosine, and Tangent

For the acute angle θ, we define the three numbers as follows

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \qquad \text{read as "sine theta"}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}, \qquad \text{read as "cosine theta"}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}, \qquad \text{read as "tangent theta"}$$

Note that these are numbers, ratios of side lengths, and have no units.

It may be convenient to enclose the argument of a trig function in parentheses. That is,

$$\sin\theta = \sin(\theta).$$

Cosecant, Secant, and Cotangent

The remaining three trigonometric functions are the reciprocals of the first three

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin\theta}, \qquad \text{read as "cosecant theta"}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos\theta}, \qquad \text{read as "secant theta"}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\tan\theta}, \qquad \text{read as "cotangent theta"}$$

A Word on Notation

The trigonometric ratios define functions:

input angle number $\rightarrow$ output ratio number.

From the definitions, we see that

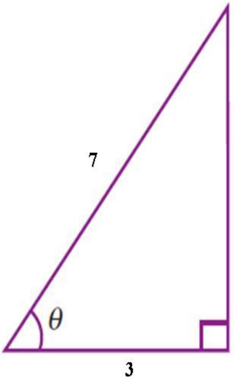
$$\csc\theta = \frac{1}{\sin\theta}.$$

Functions have arguments. It is NOT acceptable to write the above relationship as

$$\csc = \frac{1}{\sin}.$$

Example

Determine the six trigonometric values of the acute angle θ.



$a = \sqrt{40}$

Call opp, a. then

$$\sin\theta = \frac{opp}{hyp} = \frac{\sqrt{40}}{7}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{3}{7}$$

$$\tan\theta = \frac{opp}{adj} = \frac{\sqrt{40}}{3}$$

$$a^2 + 3^2 = 7^2 \Rightarrow a^2 = 7^2 - 3^2 = 40$$

$$a = \sqrt{40}$$

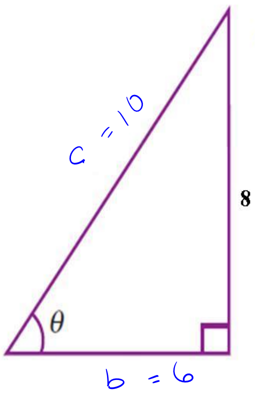
$$\csc\theta = \frac{7}{\sqrt{40}}$$

$$\sec\theta = \frac{7}{3}$$

$$\cot\theta = \frac{3}{\sqrt{40}}$$

Example

Determine the six trigonometric values of the acute angle θ.



$\sin\theta = \dfrac{4}{5}$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

Call the hypotenuse c and the adjacent leg b.

$\sin\theta = \dfrac{8}{c} = \dfrac{4}{5}$

$8 \cdot 5 = 4 \cdot c \Rightarrow c = \dfrac{8 \cdot 5}{4} = 10$

$8^2 + b^2 = 10^2 \Rightarrow b^2 = 10^2 - 8^2 = 100 - 64 = 36 \quad b = 6$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{6} = \frac{4}{3}$$

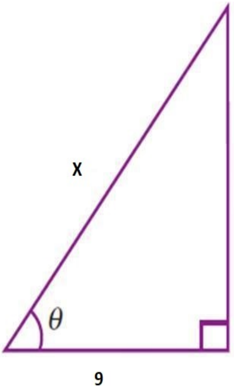
$$\csc\theta = \frac{5}{4}$$

$$\sec\theta = \frac{5}{3}$$

$$\cot\theta = \frac{3}{4}$$

Question

Suppose we know that $\cos\theta = \dfrac{3}{5}$. Then the length X of the hypotenuse



(a) $X = 5$

(b) $X = \sqrt{15}$

(c) $X = 12$

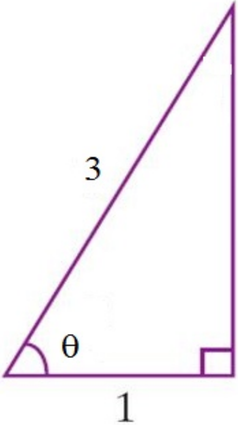
(d) $X = 15$

(e) can't be determined without more information

$$\frac{9}{X} = \frac{3}{5}$$

Question

For the angle θ shown, which statement is correct?



$\sqrt{3^2-1^2} = \sqrt{8}$

(a) $\sin\theta = \frac{\sqrt{8}}{3}$ and $\cos\theta = \frac{1}{3}$

(b) $\sin\theta = \frac{1}{3}$ and $\cos\theta = \frac{\sqrt{2}}{3}$

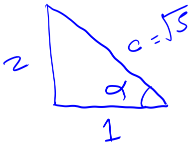
(c) $\tan\theta = \frac{1}{3}$ and $\sin\theta = \frac{\sqrt{2}}{3}$

(d) $\tan\theta = \sqrt{2}$ and $\cot\theta = \frac{1}{\sqrt{2}}$

Example

Suppose the acute angle α satisfies $\tan\alpha = 2$. Determine the remaining five trigonometric values of α.

we can create a representative triangle.



$\tan\alpha = 2 = \frac{2}{1}$

let's take opp $= 2$ and adj $= 1$

Calling the hypotenuse c

$c^2 = 2^2 + 1^2 \Rightarrow c^2 = 5 \Rightarrow c = \sqrt{5}$

$\sin\alpha = \frac{2}{\sqrt{5}}$, $\cos\alpha = \frac{1}{\sqrt{5}}$

,

$\csc\alpha = \frac{\sqrt{5}}{2}$, $\sec\alpha = \sqrt{5}$ $\cot\alpha = \frac{1}{2}$