# February 13 MATH 1112 sec. 54 Spring 2019

Section 5.5: Solving Exponential and Logarithmic Equations

**Base-Exponent Equality** For any a > 0 with  $a \neq 1$ , and for any real numbers *x* and *y* 

$$a^x = a^y$$
 if and only if  $x = y$ .

**Logarithm Equality** For and a > 0 with  $a \neq 1$ , and for any positive numbers *x* and *y* 

$$\log_a x = \log_a y$$
 if and only if  $x = y$ .

**Inverse Function** For any a > 0 with  $a \neq 1$ 

$$a^{\log_a x} = x$$
 for every  $x > 0$   
 $\log_a(a^x) = x$  for every real  $x$ .

#### Example

Find an exact solution<sup>1</sup> to the equation

 $2^{x+1} = 5^x$ 

#### We solved this equation last time and found that

$$x=\frac{\ln 2}{\ln 5-\ln 2}.$$

We used the natural log to do this, but the claim was made that any other base log could have been used.

<sup>1</sup>An exact solution may be a number such as  $\sqrt{2}$  or In(7) which requires a calculator to approximate as a decimal.

# An Observation

To solve  $2^{x+1} = 5^x$ , we used the natural log. But we have choices. Use the change of base formula to show that our solution

$$\begin{aligned}
\int hz &= \frac{\int hz}{\int hz} = \frac{\ln 2}{\ln 5 - \ln 2} = \frac{\log 2}{\log 5 - \log 2} \int_{0}^{10} \int_{0}^{10}$$

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#### Question

Jack and Diane are solving  $3^{-x} = 4^{x-1}$ . They arrive at the solutions

Jack's 
$$\frac{\ln 4}{\ln 4 + \ln 3} = \frac{\int_{0}^{0} J_{3}(4)}{\int_{0} J_{3}(4) + \int_{0}^{0} J_{3}(3)}$$
 Diane's  $\frac{\log_{3}(4)}{\log_{3}(4) + 1}$ .

Which of the following statements is true?

- (a) Jack's answer is correct, and Diane's is incorrect.
- (b) Diane's answer is correct, and Jack's is incorrect.
- (c) Both answers are correct; they are the same number.
- (d) Both answers are incorrect.

# Log Equations & Verifying Answers

Double checking answers is always recommended. When dealing with functions whose domains are restricted, **answer verification** is critical.

Use properties of logarithms to solve the equation log(x - 1) + log(x - 2) = log 12

We want to use  $\log X = \log Y \Rightarrow X = Y$ Use  $\log(mN) = \log M + \log N$   $\log(x-1) + \log(x-2) = \log 12$   $\log((x-1)(x-2)) = \log 12$ (x-1)(x-2) = 12 guadratic eqn.

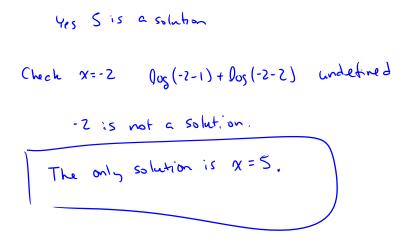
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$$X^{2} - 3x + 2 = 12$$
  
 $X^{2} - 3x - 10 = 0$   
 $(x - S)(x + 2) = 0 \implies x - S = 0 \text{ or } x + 2 = 0$   
 $x = S \text{ or } x = -2$ 

The quadratic equation has solutions 
$$S$$
 and  $-2$ ,  
we have to check if those solve the ansimal  
equation  
 $\log(x-i) + \log(x-2) = \log 12$ 

Check  $\neq = 5$  log(S-1) + log(S-2)  $log(3) = log(3\cdot 4) = log(12)$ reprive The set of the set



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