Feb. 13 Math 2254H sec 015H Spring 2015

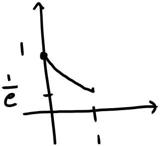
Section 7.8: Improper Integrals

Determine the convergence or divergence of the integral

$$\int_0^\infty \frac{e^{-x}}{\sqrt{x}}\,dx = \int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx + \int_1^\infty \frac{e^{-x}}{\sqrt{x}}\,dx$$

Consider the first:

$$\int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx$$



Note $\frac{1}{e} \le e^{-x} \le 1 \Rightarrow 0 \le \frac{e^{-x}}{\sqrt{x}} \le \frac{1}{\sqrt{x}}$

So $\quad 0 \le \int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx \le \int_0^1 \frac{1}{\sqrt{x}}\,dx = 2$

This integral $\int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx$ is convergent.

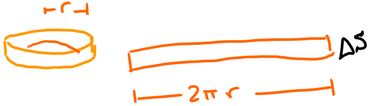
For $\int_1^\infty \frac{e^{-x}}{\sqrt{x}}\,dx \qquad 0 \le \frac{1}{\sqrt{x}} \le 1$ for all $x \ge 1$

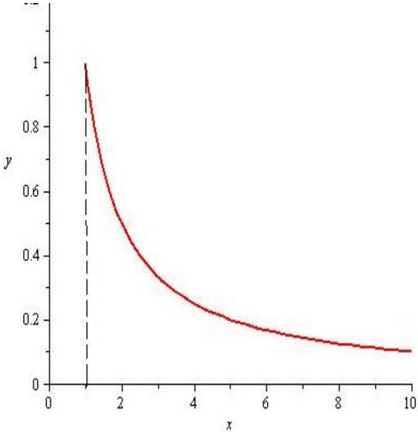
so $\quad 0 \le \frac{e^{-x}}{\sqrt{x}} \le e^{-x}$ for all $x \ge 1$

As $\int_1^\infty e^{-x}\,dx$ converges, $\int_1^\infty \frac{e^{-x}}{\sqrt{x}}\,dx$ converges.

As both pieces converge, the original integral converges.

Recall the Horn of Gabriel





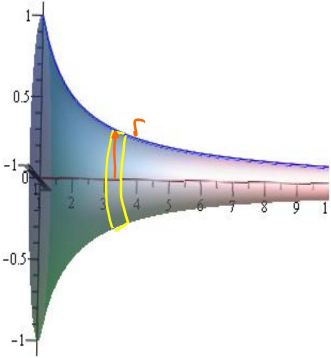


Figure: Consider the region under the curve $f(x) = \frac{1}{x}$ for $1 \leq x < \infty$. Let this be rotated about the x-axis. We found it has finite volume $V = \pi$.

Show that the Horn of Gabriel has infinite surface area

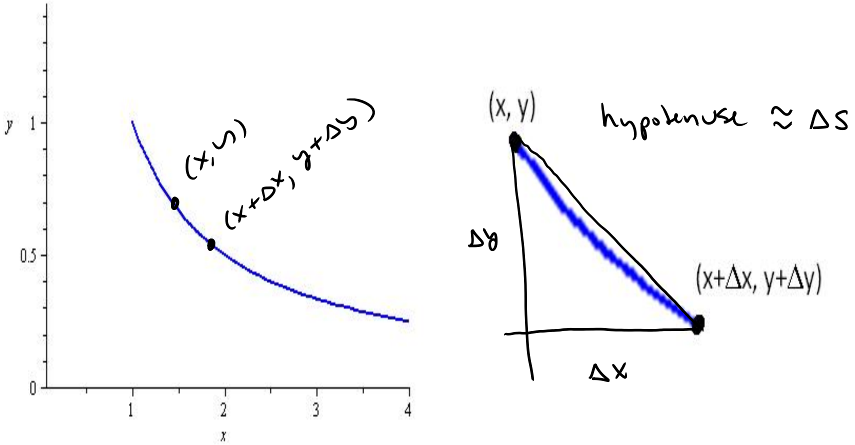


Figure: We need to characterize the length Δs of a piece of arclength for the curve.

$$\Delta s^2 \approx \Delta x^2 + \Delta y^2 = \Delta x^2 \left(1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right)$$

$$\Delta s \approx \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\ \Delta x$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

Differential arc length parameter

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$