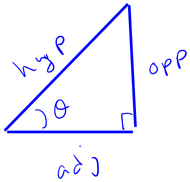
February 14 MATH 1112 sec. 54 Spring 2020

Trigonometric Functions of Acute Angles

For the acute angle θ in a right triangle with sides lengths opp, adj, and hyp, we defined the six trigonometric values of θ



$$\sin\theta = \frac{\text{opp}}{\text{hyp}},$$ read as "sine theta"

$$\cos\theta = \frac{\text{adj}}{\text{hyp}},$$ read as "cosine theta"

$$\tan\theta = \frac{\text{opp}}{\text{adj}},$$ read as "tangent theta"

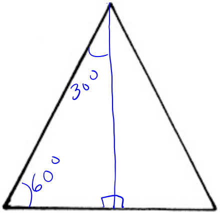
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin\theta},$$ read as "cosecant theta"

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos\theta},$$ read as "secant theta"

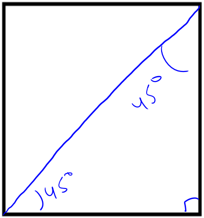
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\tan\theta},$$ read as "cotangent theta"

Some Key Trigonometric Values

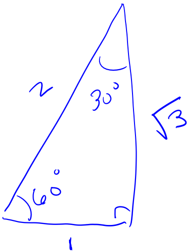
We can construct 30°-60°-90° and 45°-45°-90° right triangles.

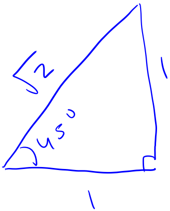


Equilateral



Square





$\sin 60^\circ = \frac{\sqrt{3}}{2}$ $\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\cos 60^\circ = \frac{1}{2}$ $\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\tan 60^\circ = \sqrt{3}$ $\tan 45^\circ = 1$

Some Key Trigonometric Values

Use the triangles to determine the six trigonometric values of the angles $30^\circ, 45^\circ$, and 60°. Those are $\frac{\pi}{6}, \frac{\pi}{4}$, and $\frac{\pi}{3}$, respectively.

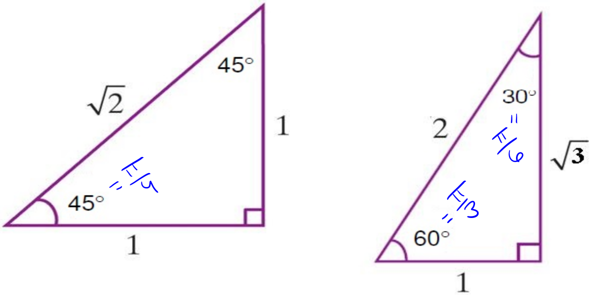


Figure: An isosceles right triangle of leg length 1 (left), and half of an equilateral triangle of side length 2 (right).

Commit To Memory

It is to our advantage to remember the following:

$$\sin 30^\circ = \frac{1}{2}, \qquad \sin 45^\circ = \frac{1}{\sqrt{2}}, \qquad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}, \qquad \cos 45^\circ = \frac{1}{\sqrt{2}}, \qquad \cos 60^\circ = \frac{1}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \qquad \tan 45^\circ = 1, \qquad \tan 60^\circ = \sqrt{3}$$

We'll use these to find some other trigonometric values. Still others will require a calculator.

Commit To Memory

These are the same trigonometric values stated in radians:

$$\sin\frac{\pi}{6} = \frac{1}{2}, \qquad \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \qquad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \qquad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \qquad \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}, \qquad \tan\frac{\pi}{4} = 1, \qquad \tan\frac{\pi}{3} = \sqrt{3}$$

We'll use these to find some other trigonometric values. Still others will require a calculator.

Question

The value $\sin 30^\circ + \tan 45^\circ =$ $\frac{1}{2} + 1 = \frac{3}{2}$

(a) 75°

(b) $\frac{2}{3}$

(c) $\frac{3}{2}$

(d) $\frac{\sqrt{3}+1}{3}$

Question

The following statements are true:

$$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2} \quad \text{and} \quad \frac{\pi}{3}=2\left(\frac{\pi}{6}\right).$$

True or False The value of $\cos\frac{\pi}{3}=2\cos\frac{\pi}{6}$.

(a) True, and I'm confident.

(b) True, but I'm not certain.

(c) False, and I'm confident.

(d) False, but I'm not certain.

$\cos\frac{\pi}{3} = \frac{1}{2}$

$2\cos\frac{\pi}{6} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

Calculator

Figure: Any scientific calculator will have built in functions for sine, cosine and tangent. (TI-84 shown)

Using a Calculator

Evaluate the following using a calculator. Round answers to three decimal places.

$\sin 16° = 0.276$

° mode

$\sec 78.3° = \dfrac{1}{\cos(78.3°)} = 4.931$

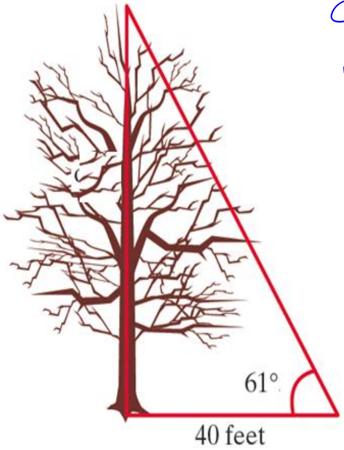
$\tan\left(\dfrac{2\pi}{7}\right) = 1.254$

radian mode

Application Example

Before cutting down a dead tree, you wish to determine its height. From a horizontal distance of 40 ft, you measure the angle of elevation from the ground to the top of the tree to be 61°. Determine the tree height to the nearest 100^{th} of a foot.

h



Call the tree height h.

Note that

$$\tan 61^\circ = \frac{h}{40 \text{ ft}}$$

$$h = 40 \tan 61^\circ \text{ ft}$$

$$\approx 72.16 \text{ ft}$$

Complementary Angles and Cofunction Identities

The two acute angles in a right triangle are complementary angles.

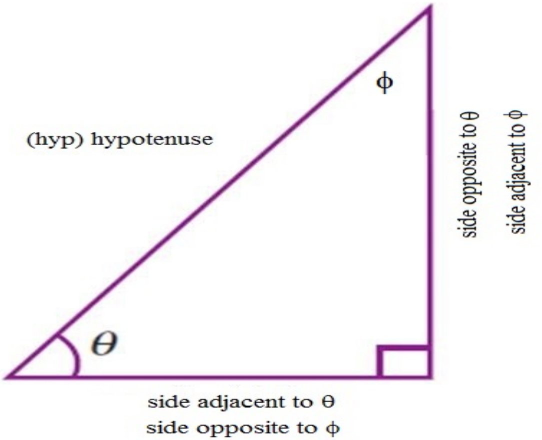


Figure: Note that for complementary angles θ and ϕ, the role of the legs (opposite versus adjacent) are interchanged.

Cofunction Identities

For any acute angle θ

$$\sin\theta = \cos(90^\circ - \theta) \qquad \cos\theta = \sin(90^\circ - \theta)$$

$$\tan\theta = \cot(90^\circ - \theta) \qquad \cot\theta = \tan(90^\circ - \theta)$$

$$\sec\theta = \csc(90^\circ - \theta) \qquad \csc\theta = \sec(90^\circ - \theta)$$

Stated in radians

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \qquad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right) \qquad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right) \qquad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

These equations define what are called **cofunction identities.**

Question

Suppose θ is an acute angle such that $\sin\theta = 0.334$. Which of the following is true?

(a) $\sin\left(\frac{\pi}{2} - \theta\right) = 1 - 0.334$

(b) $\cos\left(\frac{\pi}{2} - \theta\right) = 0.334$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

(c) $\csc\left(\frac{\pi}{2} - \theta\right) = 0.334$

(d) $\csc\theta = \dfrac{1}{1 - 0.334}$

(e) There's not enough information to determine whether any of the above is true.