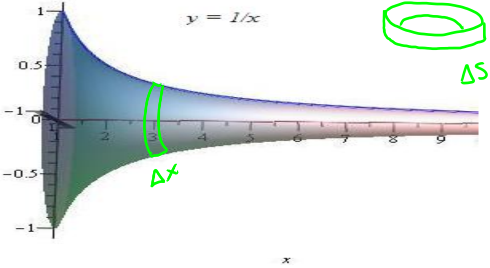
Feb. 16 Math 2254H sec 015H Spring 2015

Section 7.8: Improper Integrals

Show that the horn of Gabriel has infinite surface area:



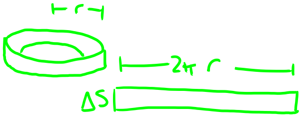


Figure: The differential arclength parameter: $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

Surface Area of a piece

$$SA_{piece} = 2\pi r \Delta s$$

$$r = \frac{1}{x} \quad \text{and} \quad \Delta s = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} \, \Delta x$$

Total Surface area

$$SA = \int_1^\infty 2\pi \frac{1}{x} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$y = \frac{1}{x}$ so $\frac{dy}{dx} = \frac{-1}{x^2} \Longrightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{x^4}$

$$SA = 2\pi \int_1^\infty \frac{1}{x}\sqrt{1 + \frac{1}{x^4}}\, dx$$

Recall $\int_1^\infty \frac{1}{x}\, dx$ is divergent.

And note that $1 + \frac{1}{x^4} \geq 1$

for all $x \geq 1$

So $1 \leq \sqrt{1+\frac{1}{x^4}} \implies$

$0 < \frac{1}{x} \leq \frac{1}{x}\sqrt{1+\frac{1}{x^4}}$ for all $x \geq 1$

Hence $\int_1^\infty \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx$ diverges

by comparison.

This mathematical object has a finite volume but an infinite surface area.

Section 10.1: Parametric Curves

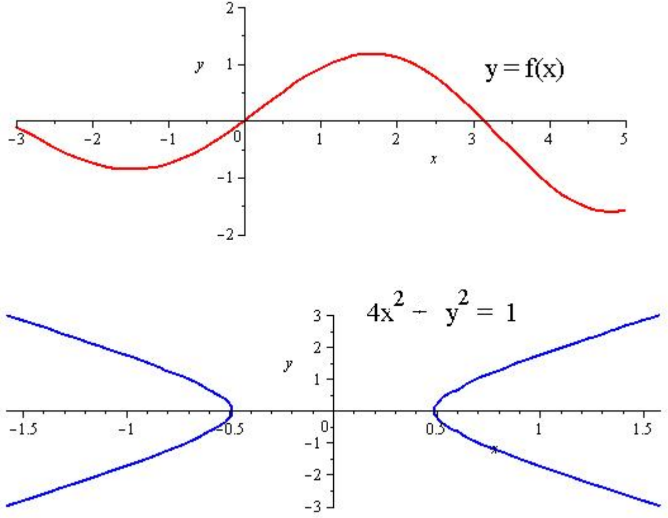


Figure: Graphs of two curves: One is an explicit function, the other is not a function but the points satisfy a relation.

Curve as a Path

Definition: We consider a **path** to be a curve along with an orientation—i.e. a specified direction of motion along that curve.

Remark: If a path is in the *xy*-plane, and a particle is traversing the path (in time), then at each moment the position of the particle can be characterized by an ordered pair

$$(x, y) = (f(t), g(t)).$$

The functions f and g dictate the value of each coordinate at each moment t in time.

Definitions: t is called a **parameter**, the pair

$$x = f(t), \quad y = g(t)$$

is called a set of **parametric equations**, and the collection of points (x, y) is called a **parametric curve**.

Example of Parametric Equations and Curve

$$x = t^2 - 2t, \quad y = t + 1, \quad 0 \le t \le 5$$

Characteristics:

- The curve (x, y) will begin at $(0, 1)$ when $t = 0$ and end at $(15, 6)$ when $t = 5$.
- The x coordinate will decrease on $0 < t < 1$ from 0 to -1, then will increase. How do we know this??[1]
- The y coordinate increases at a constant rate from 1 to 6.

[1] We'll work out the details on the following slide.

$$x = t^2 - 2t, \quad y = t + 1, \quad 0 \le t \le 5$$

$\frac{dx}{dt} = 2t - 2 = 2(t-1)$ $\qquad$ $\frac{dx}{dt} = 0$ if $t = 1$

$\frac{dx}{dt} < 0$ if $t < 1$, $\frac{dx}{dt} > 0$ if $t > 1$

$\frac{dy}{dt} = 1$

$y = t + 1 \implies t = y - 1$ so that

$x = (y-1)^2 - 2(y-1) = y^2 - 2y + 1 - 2y + 2$

$x = y^2 - 4y + 3 = (y-1)(y-3)$

for $1 \le y \le 6$

Parametric Curve[2]

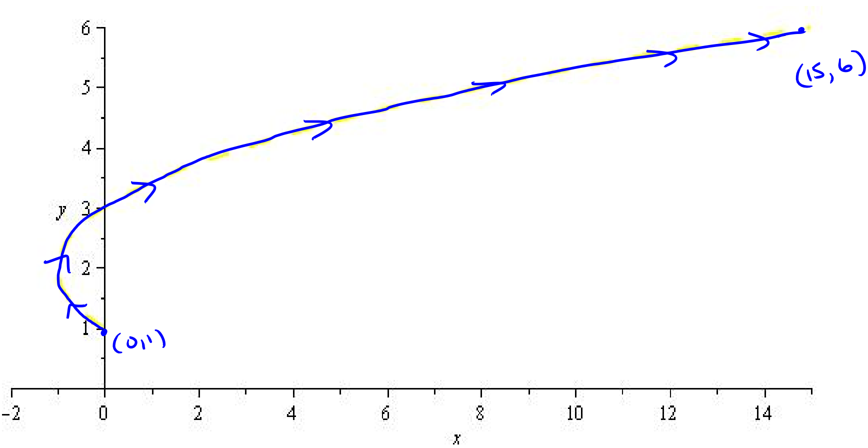


Figure: Parametric Curve $x = t^2 - 2t$, $y = t + 1$, for $0 \le t \le 5$

[2]See Maple Worksheet for particle animation.