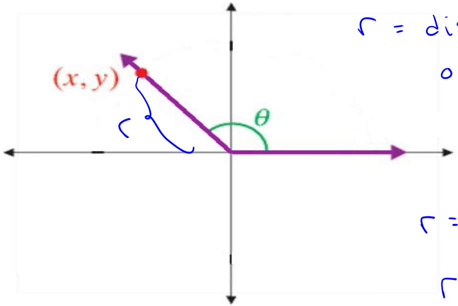
February 17 MATH 1112 sec. 52 Spring 2020

Trigonometric Functions of Any Angle

We wish to extend the definitions of the six trigonometric functions to angles that are not necessarily acute. To start, consider an angle in standard position, and choose a point (x, y) on the terminal side.



r = distance from origin of the point

$r = \sqrt{x^2 + y^2}$

$r > 0$

Trigonometric Function of any Angle

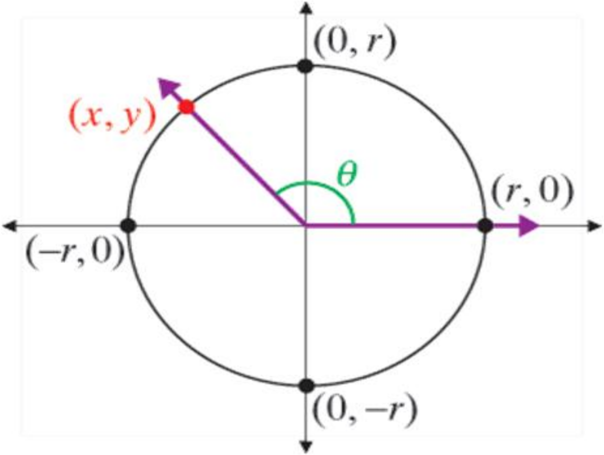


Figure: An angle in standard position determined by a point (x, y). Any such point lives on a circle in the plane centered at the origin having radius $r = \sqrt{x^2 + y^2}$

Trigonometric Function of any Angle

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x} \quad (\text{for } x \neq 0)$$

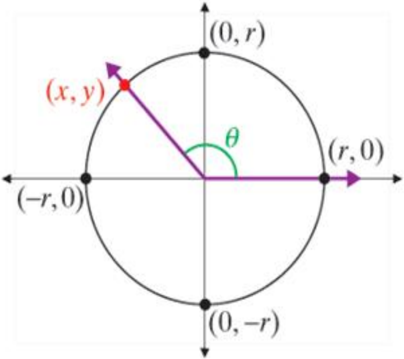


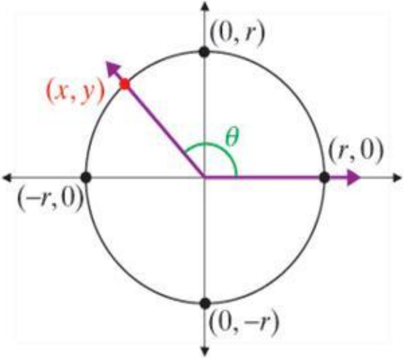
Figure: The definitions for the sine, cosine and tangent of any angle θ are given in terms of x, y, and r.

Trigonometric Function of any Angle

$$\csc\theta = \frac{r}{y} \quad (\text{for } y \neq 0)$$

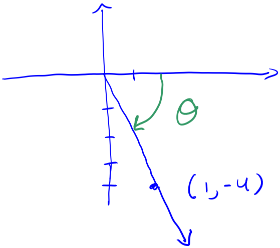
$$\sec\theta = \frac{r}{x} \quad (\text{for } x \neq 0)$$

$$\cot\theta = \frac{x}{y} \quad (\text{for } y \neq 0)$$



Example

The terminal side of an angle θ in standard position passes through the point $(1, -4)$. Determine the sine and cosine of the angle.



We need r

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-4)^2}$$

$$= \sqrt{17}$$

$$\sin\theta = \frac{y}{r} = \frac{-4}{\sqrt{17}}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{17}}$$

Also, $\tan\theta = \frac{y}{x} = \frac{-4}{1} = -4$

Question

When put in standard position, the terminal side of the angle θ passes through the point $(-2, 3)$. The sine value of θ is

$r = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$

(a) $\sin\theta = -\frac{3}{2}$

(b) $\sin\theta = \frac{3}{\sqrt{13}}$

(c) $\sin\theta = \frac{3}{5}$

(d) $\sin\theta = \frac{3}{\sqrt{5}}$

Reciprocal Identities

We have the first in a long list of **trigonometric identities**:

Reciprocal Identities: For any given θ for which both sides are defined

$$\csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \& \quad \cot\theta = \frac{1}{\tan\theta}.$$

Equivalently

$$\sin\theta = \frac{1}{\csc\theta}, \quad \cos\theta = \frac{1}{\sec\theta}, \quad \& \quad \tan\theta = \frac{1}{\cot\theta}.$$

Trigonometric Function of any Angle (Unit Circle Case)

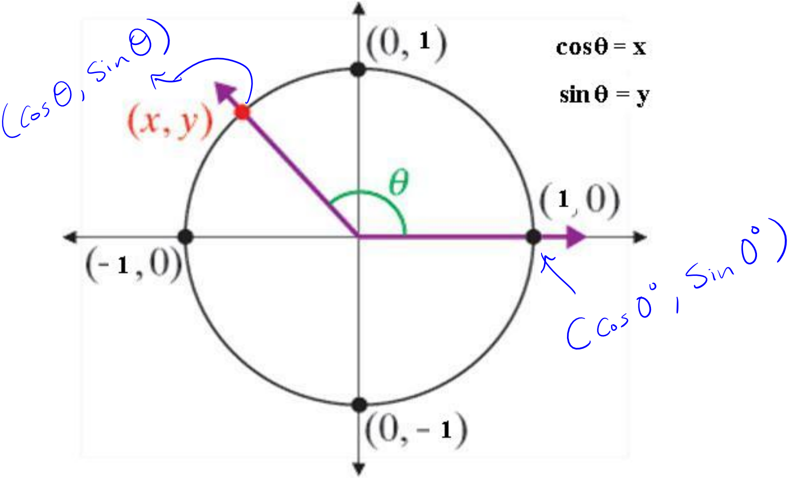


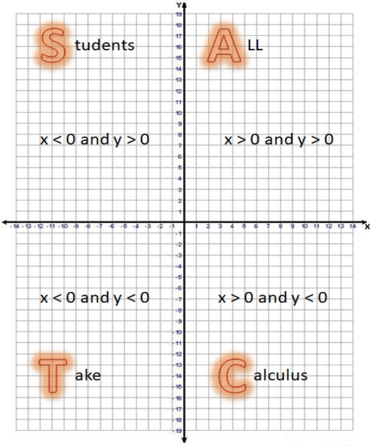
Figure: A point on the unit circle, $r = 1$, has coordinates $(x, y) = (\cos\theta, \sin\theta)$.

A Useful Table of Trigonometric Values

θ°	0°	30°	45°	60°	90°
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef.

An angle whose terminal side, when in standard position, is concurrent with one of the axes is called a **quadrantal angle**.

Quadrants & Signs of Trig Values



I: All trig values are positive.

II: Sine (cosecant) are positive.

III: Tangent (cotangent) are positive.

IV: Cosine (secant) are positive.

Figure: The trigonometric values for a general angle may be positive, negative, zero, or undefined. The signs are determined by the signs of the x and y values. **Note that $r > 0$ by definition.**

Example

Determine which quadrant the terminal side of θ must be in if

(a) $\sin\theta > 0$ and $\tan\theta < 0$

$\sin\theta > 0$ Quad I or II

$\tan\theta < 0$ Quad II or IV

$\Rightarrow$ θ's terminal side is in Quad II

(b) $\sec\theta < 0$ and $\cot\theta > 0$

$\sec\theta < 0$ Quad II or III

$\cot\theta > 0$ Quad I or III

$\Rightarrow$ θ's terminal side is in Quad III

Question

Suppose that θ is a positive angle whose measure is less than 360°, $\sin\theta = -0.3420$, and $\cos\theta = -0.9397$. Which of the following must be true about θ?

$\sin\theta < 0$ and $\cos\theta < 0$

(a) $0° < \theta < 90°$

(b) $90° < \theta < 180°$

(c) $180° < \theta < 270°$

(d) $270° < \theta < 360°$

(e) any of the above may be true, more information is needed to determine which is true