February 25 MATH 1112 sec. 54 Spring 2019

Radian Measure (Some section 6.4)

We defined radian measure, and can convert between radian and degree angle measures.

Converting Between Degrees & Radians

Since $360^\circ = 2\pi$ rad, we get the following conversion factors:

$$1^\circ = \frac{\pi}{180} \text{ rad} \qquad \text{and} \qquad 1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ$$

Remark: If an angle doesn't have the degree symbol $^\circ$ next to it, it is assumed to be in radians!

Question

The radian equivalent to 20° is

(a) $\dfrac{360}{\pi}$

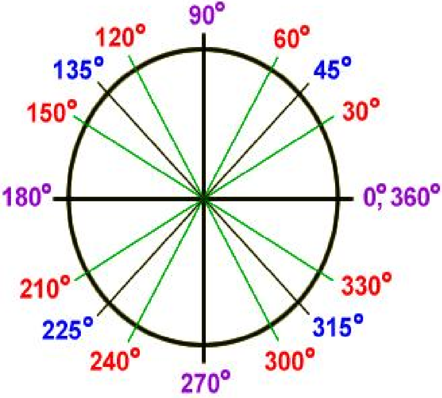
(b) $\dfrac{\pi}{10}$

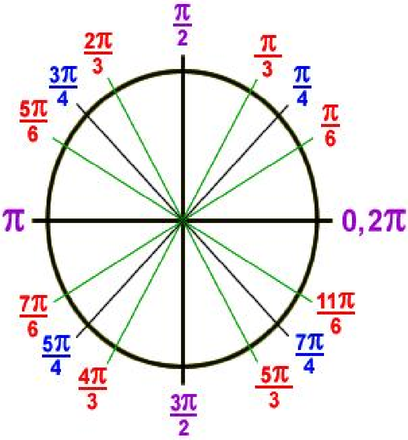
(c) 20π

(d) $\dfrac{\pi}{9}$

$$20 \cdot \frac{\pi}{180} = \frac{2\pi}{18} = \frac{\pi}{9}$$

Some common angles in both measures...



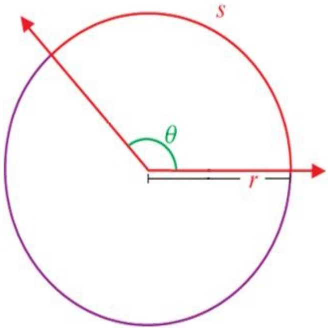


Arclength Formula

Given a circle of radius r, the length s of the arc subtended by the (positive) central angle θ (**in radians**) is given by

$$s = r\theta.$$

The area of the resulting sector is $A_{sector} = \frac{1}{2}r^2\theta$.



Question

$s = r\theta$, $A_{sector} = \frac{1}{2} r^2 \theta$

An industrial clock has a face that is 3 ft in **diameter.** What is the area of the sector between the 12 and the 4 hour markings?

(a) $\frac{9\pi}{2}$ ft^2

(b) $\frac{3\pi}{2}$ ft^2

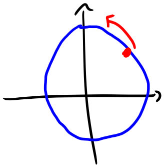
(c) $\frac{3\pi}{4}$ ft^2

(d) 3π ft^2

(e) can't be determined without more information

Motion on a Circle: Angular & Linear Speed

Definition: (angular speed) If an object moves along the arc of a circle through a central angle θ in the time t, the angular speed is denoted by ω (lower case omega) and is defined by



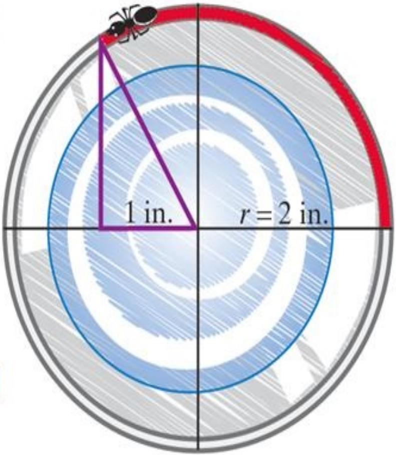
$$\omega = \frac{\theta}{t}.$$

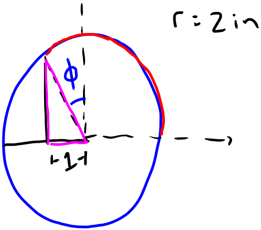
Definition: (linear speed) If the circle has radius r, then the distance traveled is the arclength $s = r\theta$. The linear speed is denoted by ν (lower case nu) and is defined by

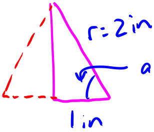
$$\nu = \frac{s}{t} = \frac{r\theta}{t} = r\omega.$$

Example

Suppose an ant crawls along the rim of a circular glass with radius 2 inches, and traverses the arc indicated in red in 20 seconds. What are the angular and linear speeds of the ant, and how far does it travel?







angle in an equilateral triangle $= \frac{360^\circ}{3} = 60^\circ$

$$\phi + 60^\circ = 90^\circ \Rightarrow \phi = 30^\circ$$

The ant traveled through $\theta = 90^\circ + 30^\circ = 120^\circ$

In radians, $\theta = 120^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{3}$

We have $\theta = \frac{2\pi}{3}$, $r = 2\text{in}$, and the time interval $t = 20\text{ sec}$

The angular speed $\omega = \frac{\theta}{t} = \frac{2\pi/3}{20\text{ sec}} = \frac{\pi}{30}\,\frac{1}{\text{sec}}$

The linear speed $v = r\omega = (2\text{in})\cdot \frac{\pi}{30}\,\frac{1}{\text{sec}}$

$$= \frac{\pi}{15}\,\frac{\text{in}}{\text{sec}}$$

The distance traveled $s = r\theta = (2\text{in})\left(\frac{2\pi}{3}\right) = \frac{4\pi}{3}\text{ in}$

Caveat!

Remember that the formulas for

arclength, sector area, angular speed, & linear speed

are for an angle in radians. An angle in degrees must be converted to radians before applying any of these formulas.