February 26 MATH 1112 sec. 52 Spring 2020

Trigonometric Functions
Graphs of Sine and Cosine Functions

Our goal is to graph functions of the form

$$f(x) = a\sin(bx - c) + d \quad \text{or} \quad f(x) = a\cos(bx - c) + d$$

Note: here we will be graphing points (x, y) on a curve $y = f(x)$.

Amplitude

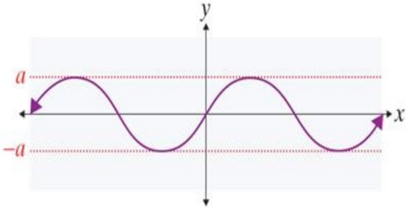
Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

Definition: Let a be any nonzero real number. The **amplitude** of the function f defined above is the value $|a|$.

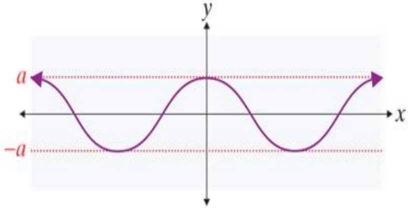
Recall that this is half the distance between the maximum and minimum values.

If $a < 0$ the graph is reflected in the x-axis. But the amplitude is still $|a|$.

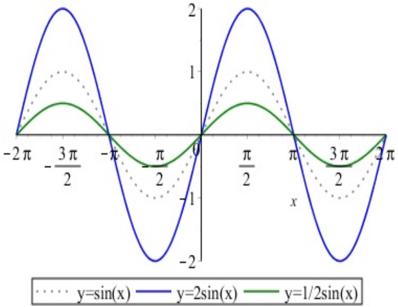
Amplitude

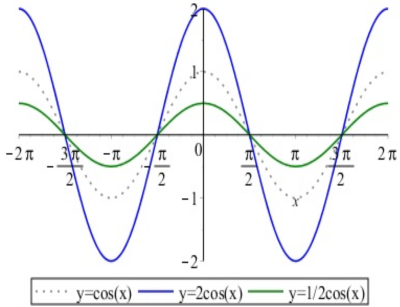


$f(x) = a \sin x$



$f(x) = a \cos x$





Example

Identify the amplitude of each function. Determine if the graph is reflected in the x-axis.

(a) $f(x) = 3\sin(4x-2)+1$

$a = 3$

Amplitude $= |3| = 3$

$a > 0$ no reflection

(b) $f(x) = 2-6\cos(2x+3)$

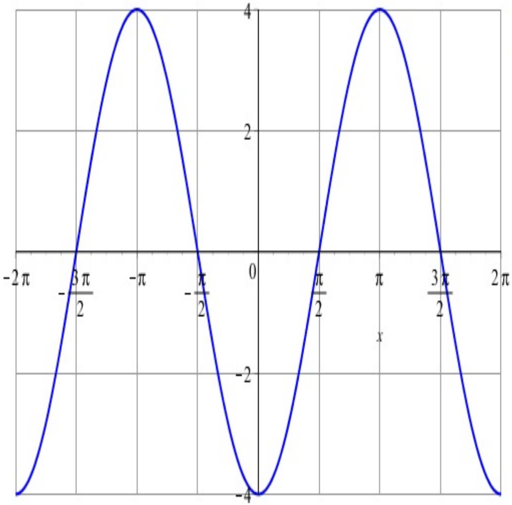
$= -6\cos(2x+3)+2$

$a = -6$

Amplitude $= |-6| = 6$

$a < 0$ horizontal reflection

Question



The figure shows two periods of the plot of $y = f(x)$ where

(a) $f(x) = 4\sin x$

(b) $f(x) = -4\sin x$

(c) $f(x) = 4\cos x$

(d) $f(x) = -4\cos x$

Period

Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

Theorem: Let b be any positive real number. The **fundamental period** of the function f above is given by

$$T = \frac{2\pi}{b}.$$

Due to symmetry, we can always assume $b > 0$. Allowing b to be signed, the period would be written as

$$T = \frac{2\pi}{|b|}.$$

Period

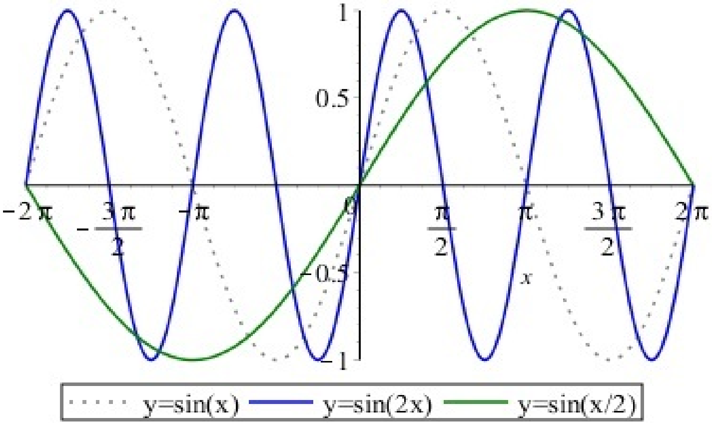


Figure: Comparisons with $b = 1/2, 1,$ and 2. On the interval $-2\pi < x < 2\pi$ we obtain one ($b = 1/2$), two ($b = 1$) or four ($b = 2$) full cycles.

Period

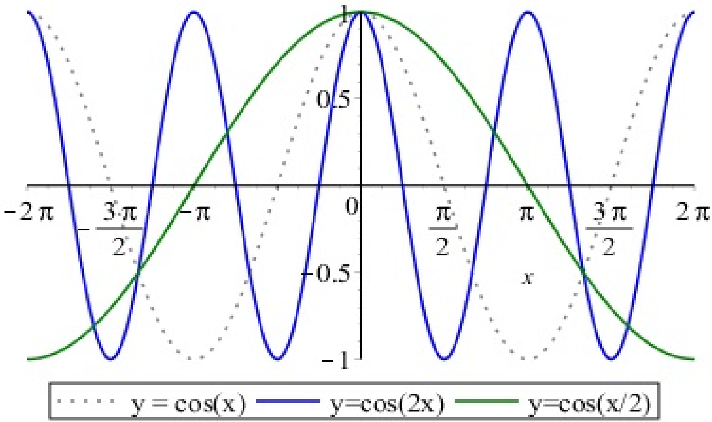


Figure: Comparisons with $b = 1/2, 1$, and 2. On the interval $-2\pi < x < 2\pi$ we obtain one ($b = 1/2$), two ($b = 1$) or four ($b = 2$) full cycles.

Example

Identify the period of each function.

(a) $f(x) = 3\sin(4x-2)+1$

$b = 4$

Period $T = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

(b) $f(x) = 2-6\cos(2x+3)$

$= -6\cos(2x+3)+2$

$b = 2$

$T = \frac{2\pi}{2} = \pi$

Frequency

Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

Definition: The reciprocal of the period is called the **frequency**. That is

$$\text{frequency} = \frac{1}{T} = \frac{b}{2\pi}.$$

If x represents time, then

- the period tells us how much time is required for one full cycle, and
- the frequency tells us how many cycles occur in one time unit.

If $y = \cos(bx)$ (or $y = \sin(bx)$), then b the number of cycles occuring in an interval of length 2π.

Question

The period of $y = 2\cos\left(\frac{\pi x}{2}\right)$ is

(a) $T = \frac{\pi}{2}$

$b = \frac{\pi}{2}$

$T = \frac{2\pi}{b} = \frac{2\pi}{\pi/2}$

$= 4$

(b) $T = 4\pi$

(c) $T = 4$

(d) $T = 2\pi^2$

Phase Shift (horizontal shift)

Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

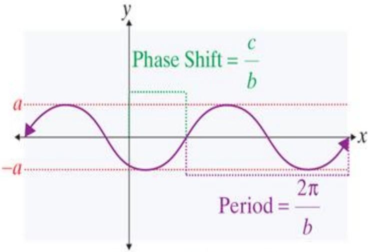
Definition: A horizontal shift is called a **phase shift.** Again assuming that $b > 0$, the phase shift for f above is

$$\frac{|c|}{b} \quad \text{units}$$

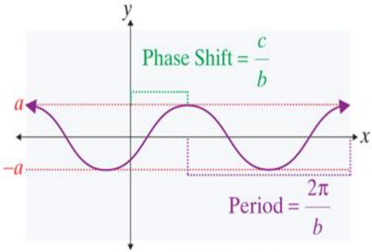
$bx - c$

$= b\left(x - \frac{c}{b}\right)$

to the right if $c > 0$ and to the left if $c < 0$.



$f(x) = a\sin(bx - c)$

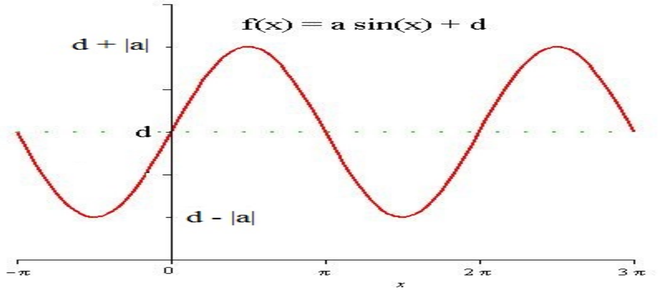


$f(x) = a\cos(bx - c)$

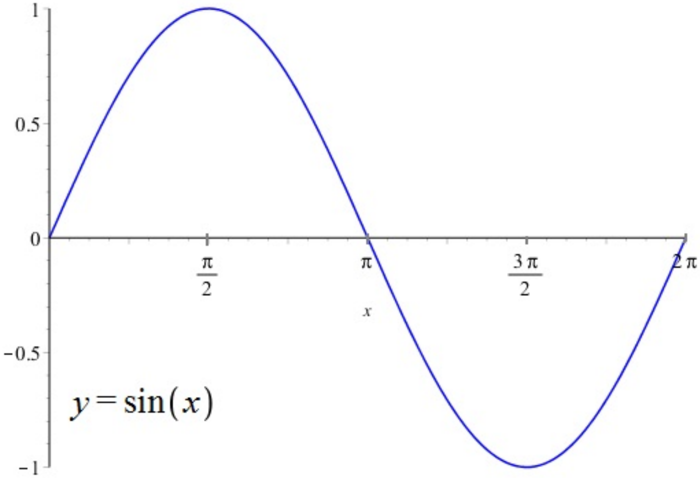
Vertical Shift

Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

Definition: If d is a nonzero number, then the function f has a **vertical shift** of $|d|$ units up if $d > 0$ and down if $d < 0$.



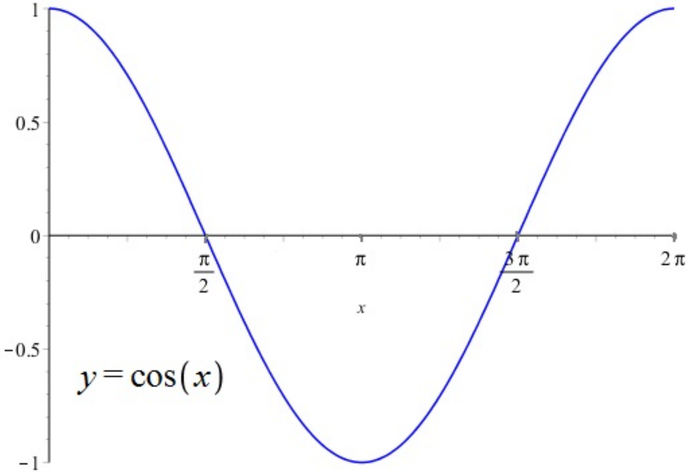
Parent Plots



The period can be divided into four equal segments.

For the sine function x-int $\rightarrow$ max $\rightarrow$ x-int $\rightarrow$ min $\rightarrow$ x-int

Parent Plots



The period can be divided into four equal segments.

For the cosine function max → x-int → min → x-int → max

$$f(x) = 2 - 4\cos\left(\pi x - \frac{\pi}{2}\right) = -4\cos\left(\pi x - \frac{\pi}{2}\right) + 2$$

Identify the period, amplitude, phase shift, any vertical shift, and reflection. Identify the key points of a full period. Plot two full periods.

$a = -4$

$b = \pi$

$c = \frac{\pi}{2}$

$d = 2$

$$\frac{|c|}{b} = \frac{\pi/2}{\pi} = \frac{1}{2}$$

- Amplitude $|a| = |-4| = 4$
- Period $\frac{2\pi}{b} = \frac{2\pi}{\pi} = 2$
- Phase shift $|c|/b = \frac{1}{2}$ right shift $\frac{1}{2}$ units
- Vertical shift 2 up
- Horizontal reflection? yes since $a < 0$

$$f(x) = 2 - 4\cos\left(\pi x - \frac{\pi}{2}\right)$$

Key points:

phase shift $\frac{1}{2}$ right

period of 2

$\frac{\text{period}}{4} = \frac{2}{4} = \frac{1}{2}$

x	$f(x)$
$\frac{1}{2}$	-2
1	2
$\frac{3}{2}$	6
2	2
$\frac{5}{2}$	-2

Key points for $y = \cos x$

x	$\cos x$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

$f(x) = 2 - 4\cos\left(\pi x - \frac{\pi}{2}\right)$

Plot two periods of its graph.