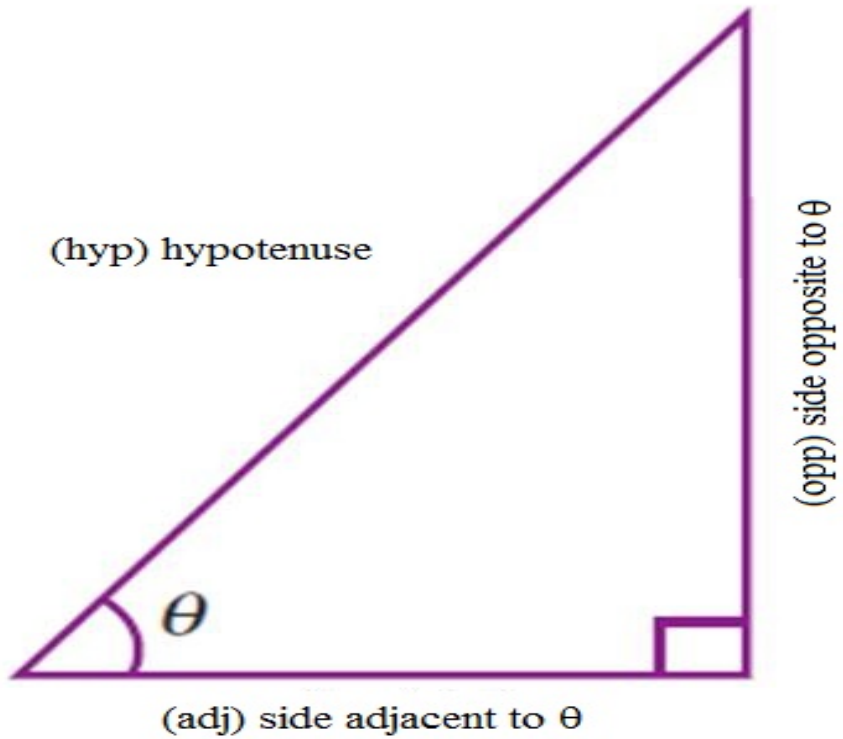
February 27 MATH 1112 sec. 54 Spring 2019

Sections 6.1 & 6.2: Trigonometric Functions of Acute Angles

In this section, we are going to define six new functions called **trigonometric functions**. We begin with an acute angle θ in a right triangle with the sides whose lengths are labeled:



Sine, Cosine, and Tangent

For the acute angle θ, we define the three numbers as follows

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \qquad \text{read as "sine theta"}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}, \qquad \text{read as "cosine theta"}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}, \qquad \text{read as "tangent theta"}$$

Note that these are numbers, ratios of side lengths, and have no units.

It may be convenient to enclose the argument of a trig function in parentheses. That is,

$$\sin\theta = \sin(\theta).$$

Cosecant, Secant, and Cotangent

The remaining three trigonometric functions are the reciprocals of the first three

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin\theta}, \qquad \text{read as "cosecant theta"}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos\theta}, \qquad \text{read as "secant theta"}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\tan\theta}, \qquad \text{read as "cotangent theta"}$$

A Word on Notation

The trigonometric ratios define functions:

$$\text{input angle number} \to \text{output ratio number.}$$

From the definitions, we see that

$$\csc\theta = \frac{1}{\sin\theta}.$$

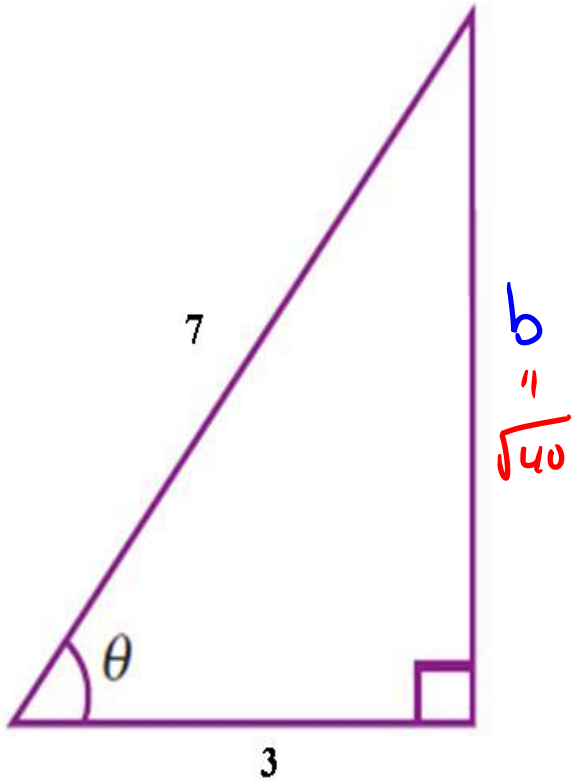
Functions have arguments. It is NOT acceptable to write the above relationship as

$$\csc = \frac{1}{\sin}.$$

Writing sin is like writing $\sqrt{\ }$ with nothing under the sign.

Example

Determine the six trigonometric values of the acute angle θ.



If the opposite leg is b, then

$3^2 + b^2 = 7^2$

$b^2 = 7^2 - 3^2 = 40 \Rightarrow b = \sqrt{40} = 2\sqrt{10}$

$\sin\theta = \frac{\sqrt{40}}{7}$ $\quad \frac{opp}{hyp}$

$\cos\theta = \frac{3}{7}$ $\quad \frac{adj}{hyp}$

$\tan\theta = \frac{\sqrt{40}}{3}$ $\quad \frac{opp}{adj}$

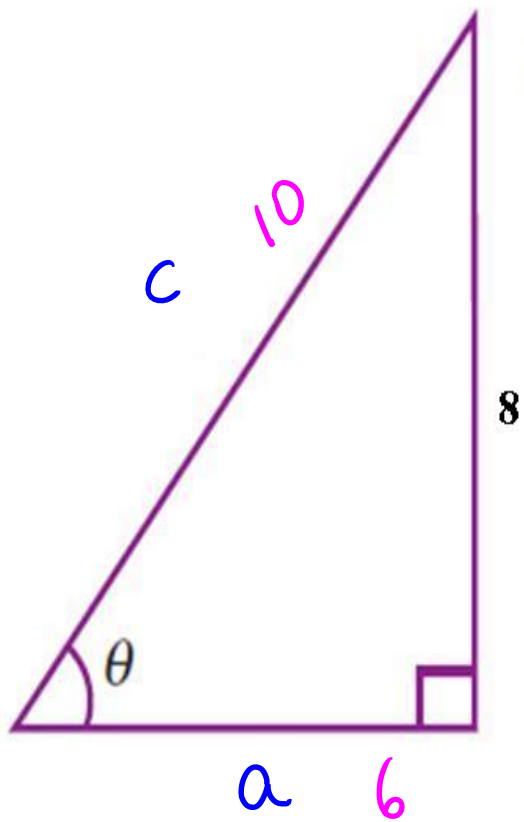
$$\csc\theta = \frac{1}{\sin\theta} = \frac{7}{\sqrt{40}} \quad \left(= \frac{7\sqrt{40}}{40}\right)$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{7}{3}$$

$$\cot\theta = \frac{3}{\sqrt{40}}$$

Example

Determine the six trigonometric values of the acute angle θ.



$\sin\theta = \frac{4}{5}$

If the hypotenuse is c

then $\sin\theta = \frac{8}{c}$.

$\frac{8}{c} = \frac{4}{5} \Rightarrow 8 \cdot 5 = 4c$

$c = 10$

Calling the adjacent leg a

$a^2 + 8^2 = 10^2 \Rightarrow a^2 = 100 - 64 = 36$

$a = 6$

adj = 6, opp = 8, and hyp = 10

$\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{6}{10} = \frac{3}{5}$

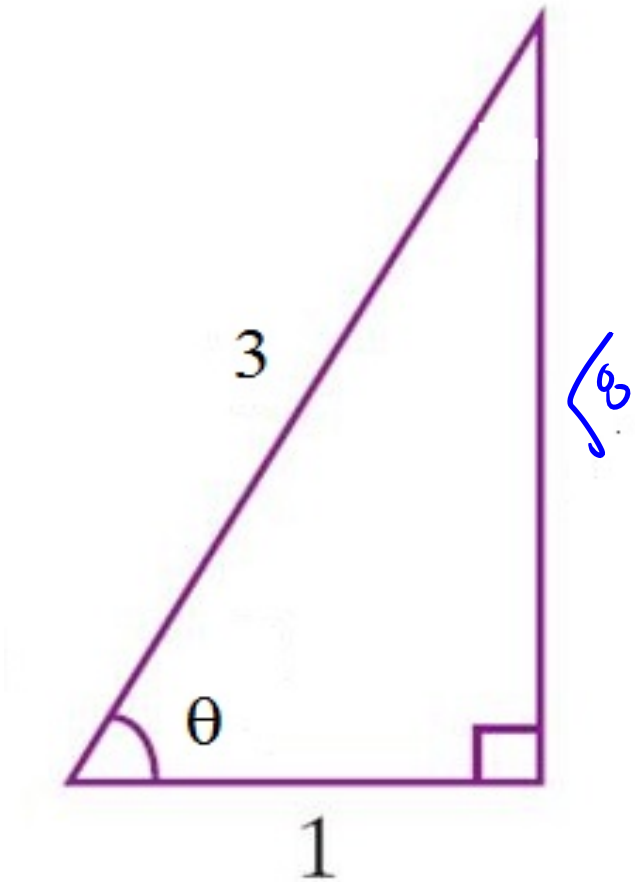
$\tan\theta = \frac{8}{6} = \frac{4}{3}$, $\cot\theta = \frac{6}{8} = \frac{3}{4}$

$\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$

$\csc\theta = \frac{1}{\sin\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$

Question

For the angle θ shown, which statement is correct?



(a) $\sin\theta = \frac{\sqrt{8}}{3}$ and $\cos\theta = \frac{1}{3}$

(b) $\sin\theta = \frac{1}{3}$ and $\cos\theta = \frac{\sqrt{2}}{3}$

(c) $\tan\theta = \frac{1}{3}$ and $\sin\theta = \frac{\sqrt{2}}{3}$

(d) $\tan\theta = \sqrt{2}$ and $\cot\theta = \frac{1}{\sqrt{2}}$

Question

(True or False) If $\sin = \dfrac{1}{\sqrt{3}}$, then $\cos = \dfrac{\sqrt{2}}{\sqrt{3}}$.

- True, because $1^2 + (\sqrt{2})^2 = (\sqrt{3})^2$ (a)
- False because neither of those equations make any sense. (b)

"sin" isn't a thing (in math)

These functions need an argument.