February 26 MATH 1112 sec. 52 Spring 2020

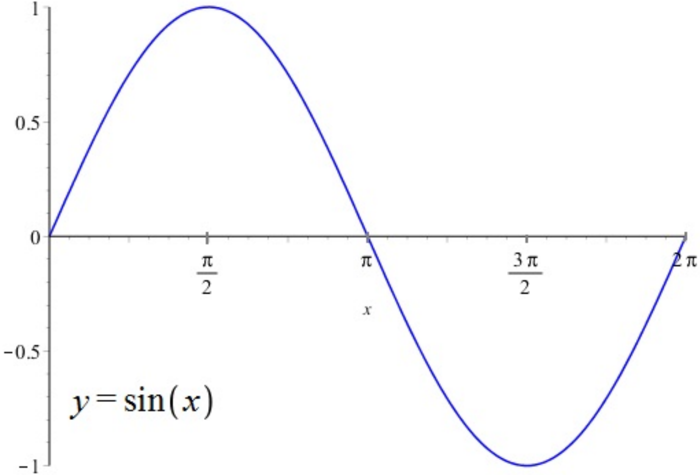
Trigonometric Functions
Graphs of Sine and Cosine Functions

Our goal is to graph functions of the form

$$f(x) = a\sin(bx - c) + d \quad \text{or} \quad f(x) = a\cos(bx - c) + d$$

- Amplitude $= |a|$
- Period $T = \dfrac{2\pi}{b}$
- Phase shift (horizontal) is $\dfrac{|c|}{b}$ (right if $c > 0$ and left if $c < 0$)
- Vertical shift is d up if $d > 0$ and down if $d < 0$

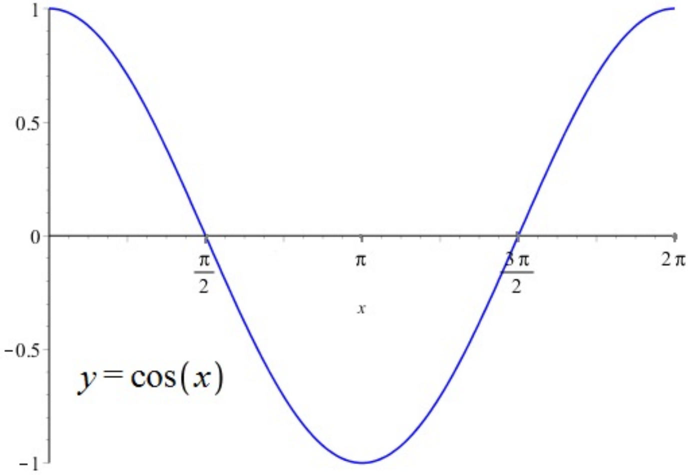
Parent Plots



The period can be divided into four equal segments.

For the sine function x-int $\to$ max $\to$ x-int $\to$ min $\to$ x-int

Parent Plots



The period can be divided into four equal segments.

For the cosine function max → x-int → min → x-int → max

The Tangent

The function $\tan s = \dfrac{\sin s}{\cos s}$. Recall that

$$\cos s = 0 \quad \text{whenever} \quad s = \frac{m\pi}{2} \quad \text{for} \quad m = \pm 1, \pm 3, \pm 5, \ldots$$

When $\cos s = 0$, $\sin s$ is either 1 or -1. Hence

Domain: The domain of the tangent function is all real number **except** odd multiples of $\pi/2$. We can write this as

$$\left\{ s \;\middle|\; s \neq \frac{\pi}{2} + k\pi,\ k = 0, \pm 1, \pm 2, \ldots \right\}$$

Moreover, the graph of the tangent function has vertical asymptotes at each odd multiple of $\pi/2$.

The Tangent

Range: The range of the tangent function is **all real numbers.**

Symmetry: The function $f(s) = \tan s$ is odd. That is

$$f(-s) = \tan(-s) = -\tan s = -f(s).$$

Perodicity: The tangent function is periodic with fundamental period π. That is

$$\tan(s + \pi) = \tan s \quad \text{for all } s \text{ in the domain.}$$

Note: The period of the tangent function is π. This is different from the period of the sine and cosine.

The Tangent

A few key tangent values:

s	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan s$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef.

And due to symmetry

s	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0
$\tan s$	undef.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Here is an applet to plot two periods of the function $f(s) = \tan s$.

GeoGebra Graph Applet: Tangent

Basic Plot $f(x) = \tan x$

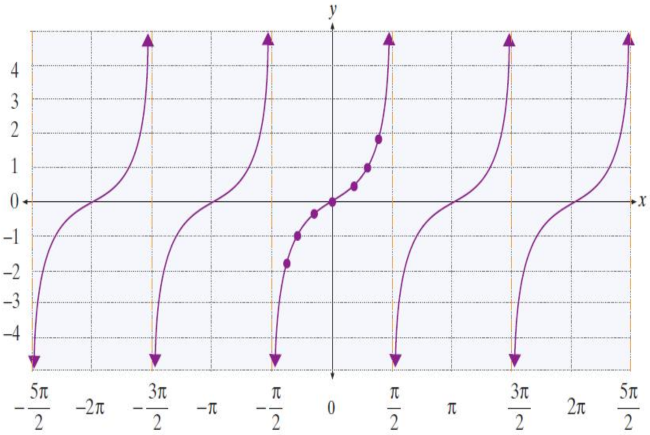


Figure: Plot of several periods of $f(x) = \tan x$. Note that the interval between adjacent asymptotes is the period π.

Cotangent: Using the Cofunction ID and Symmetry

$$\cot s = \tan\left(\frac{\pi}{2} - s\right) = \tan\left(-\left(s - \frac{\pi}{2}\right)\right) = -\tan\left(s - \frac{\pi}{2}\right).$$

So the graph of $f(s) = \cot s$ is the graph of $g(s) = \tan s$ under a horizontal shift $\pi/2$ units to the right followed by a reflection in the s-axis.

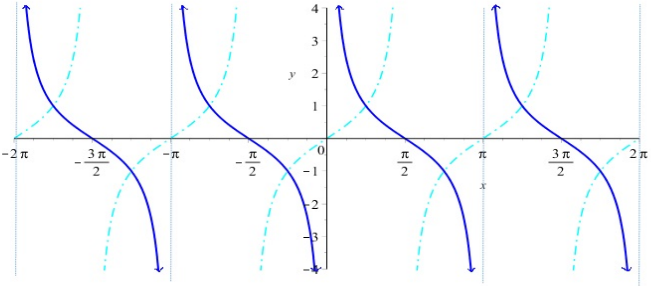


Figure: Plot of $f(x) = \cot x$. Note that the lines $x = n\pi$ for $n = 0, \pm 1, \pm 2, \ldots$ are vertical asymptotes to the graph. The dashed curve is $y = \tan x$.

Cosecant and Secant

Domains: Since $\sin(n\pi) = 0$ for integers n,

$$\textbf{Domain}(\csc s) \;=\; \{s \mid s \neq n\pi,\ \text{for integers } n\}.$$

Since $\cos\left(\frac{\pi}{2} + n\pi\right) = 0$ for integers n, the domain of $\sec s$ is

$$\textbf{Domain}(\sec s) \;=\; \left\{s \;\middle|\; s \neq \frac{\pi}{2} + n\pi,\ \text{for integers } n\right\}.$$

Ranges: Note that

$$|\csc s| = \frac{1}{|\sin s|} \geq 1 \quad \text{and} \quad |\sec s| = \frac{1}{|\cos s|} \geq 1$$

so the range of both $\csc s$ and $\sec s$ is

$$(-\infty, -1] \cup [1, \infty).$$

Cosecant: Using $\csc s = \dfrac{1}{\sin s}$

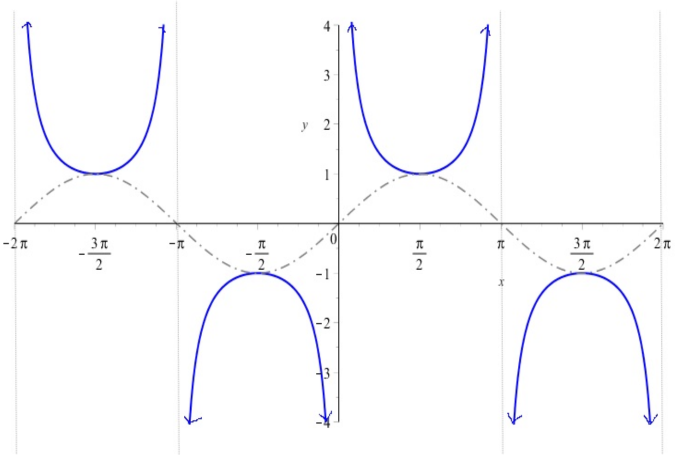


Figure: Two periods of $f(s) = \csc s$. The dashed curve is $y = \sin s$. Note the asymptotes $s = n\pi$ for integers n where $\sin s$ takes its zeros. The curves meet at the relative extrema and have the same period 2π.

Secant: Using $\sec s = \frac{1}{\cos s}$

$\frac{1}{1} = 1$

$\frac{1}{-1} = -1$

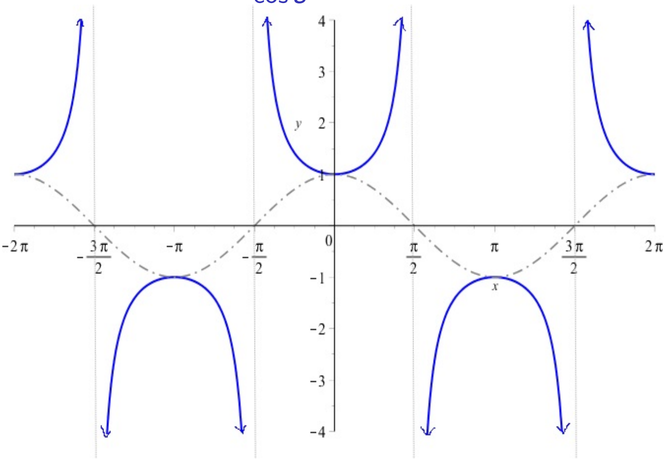


Figure: Two periods of $f(s) = \sec s$. The dashed curve is $y = \cos s$. Note the asymptotes $s = \pi/2 + n\pi$ for integers n where $\cos s$ takes its zeros. The curves meet at the relative extrema and have the same period 2π.

Example

$y = a\sin(bx - c) + d$

Analyze and plot $y = 2\sin(2x) - 1$

$a = 2$
$b = 2$
$c = 0$
$d = -1$

Amplitude $= |2| = 2$

$|a|$

Period $T = \frac{2\pi}{2} = \pi$

$\frac{2\pi}{b}$

Phase shift none $\frac{|c|}{b}$

Vertical shift down 1 d down since $d < 0$

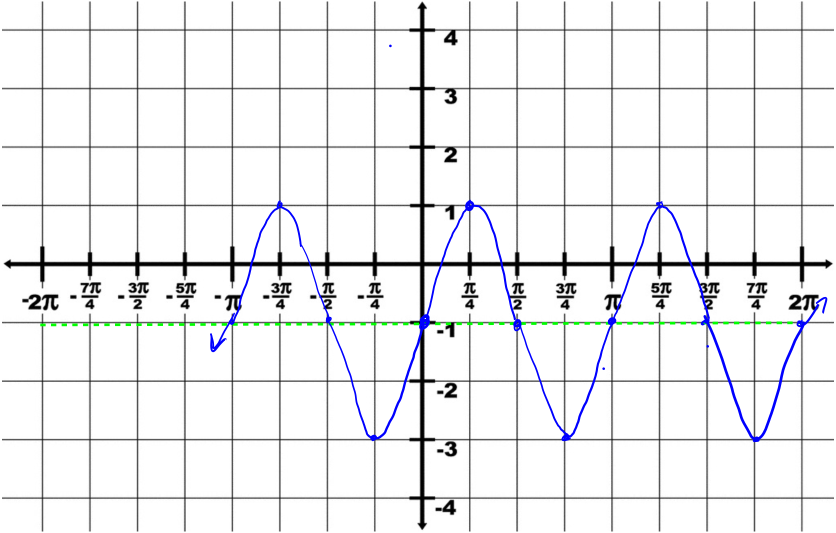
$y = 2\sin(2x) - 1$

$c = 0$ $T = \pi$

1/4 period is $\frac{\pi}{4}$

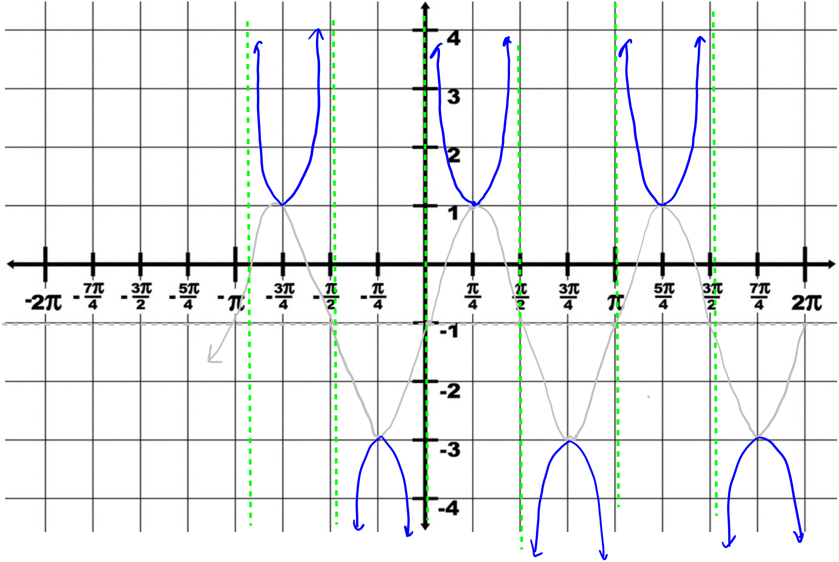
x	$2\sin(2x) - 1$
0	$2 \cdot 0 - 1 = -1$
$\pi/4$	$2 \cdot 1 - 1 = 1$
$\frac{\pi}{2}$	$2 \cdot 0 - 1 = -1$
$\frac{3\pi}{4}$	$2 \cdot (-1) - 1 = -3$
π	$2 \cdot 0 - 1 = -1$

x	$\sin x$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0


4
3
2
1
-1
-2
-3
-4
-2π
-7π/4
-3π/2
-5π/4
-π
-3π/4
-π/2
-π/4
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
2π

Example

Analyze and plot $y = 2\csc(2x) - 1$


4
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-1
-2
-3
-4
-2π
-7π/4
-3π/2
-5π/4
-π
-3π/4
-π/2
-π/4
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
2π