Feb. 5 Math 2254H sec 015H Spring 2015

Section 7.5: Strategies for Integrating

Techniques for Integrating

- By observation (recognition from differentiation rules)
- u-Substitution
- Integration by Parts
- Trigonometric functions (using trigonometric IDs)
- Trigonometric Substitution (for radicals like $\sqrt{\pm x^2 \pm a^2}$)
- Rational Functions (partial fractions)

These can be combined with one another and with algebra and function identities.

Evaluate Each Integral Using Any Applicable Method

(a) $\int_0^1 \sqrt{t}(t^2+2)\,dt$ $=$ $\int_0^1 \left(t^{5/2} + 2t^{1/2}\right) dt$

$$= \frac{2}{7} t^{7/2} + \frac{4}{3} t^{3/2} \Big|_0^1$$

$$= \frac{2}{7} + \frac{4}{3} - 0 = \frac{6+28}{21} = \frac{34}{21}$$

Evaluate Each Integral Using Any Applicable Method

(b) $\int 16x \sin x \cos x \, dx = \int 8x \sin(2x)\, dx$

$u = 8x \qquad du = 8\,dx$

$v = -\frac{1}{2}\cos 2x \qquad dv = \sin 2x \, dx$

$= -4x\cos 2x + \int 4\cos 2x \, dx$

$= -4x\cos 2x + 2\sin 2x + C$

Evaluate Each Integral Using Any Applicable Method

(c) $\displaystyle\int \frac{d\theta}{1-\sin\theta}$ $\displaystyle = \int \frac{1}{1-\sin\theta}\cdot\left(\frac{1+\sin\theta}{1+\sin\theta}\right)d\theta$

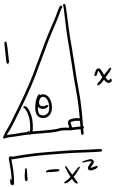
$$= \int \frac{1+\sin\theta}{\cos^2\theta}\,d\theta = \int \frac{1}{\cos^2\theta}\,d\theta + \int \frac{\sin\theta}{\cos^2\theta}\,d\theta$$

$$= \int \sec^2\theta\,d\theta + \int \sec\theta\tan\theta\,d\theta$$

$$= \tan\theta + \sec\theta + C$$

Evaluate Each Integral Using Any Applicable Method

(d) $\displaystyle\int_0^{1/\sqrt{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx$



$$= \int_0^{\pi/4} \frac{\sin^2\theta \, \cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}}$$

$$= \int_0^{\pi/4} \sin^2\theta \; d\theta$$

$x = \sin\theta$

$dx = \cos\theta \, d\theta$

$\sqrt{1-x^2} = \cos\theta$

$x = 0 \quad \sin\theta = 0 \Rightarrow \theta = 0$

$x = \frac{1}{\sqrt{2}} \quad \sin\theta = \frac{1}{\sqrt{2}} \quad \theta = \frac{\pi}{4}$

$$= \int_0^{\pi/4} \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta \right) d\theta$$

$$= \frac{\theta}{2} - \frac{1}{4}\sin 2\theta \Big|_0^{\pi/4}$$

$$= \frac{\pi/4}{2} - \frac{1}{4}\sin\frac{\pi}{2} - 0 = \frac{\pi}{8} - \frac{1}{4}$$