Feb. 6 Math 2254H sec 015H Spring 2015

Section 7.5: Strategies for Integrating

Techniques for Integrating

- By observation (recognition from differentiation rules)
- u-Substitution
- Integration by Parts
- Trigonometric functions (using trigonometric IDs)
- Trigonometric Substitution (for radicals like $\sqrt{\pm x^2 \pm a^2}$)
- Rational Functions (partial fractions)

These can be combined with one another and with algebra and function identities.

Some Useful Trigonometric Identities

Recall

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

and

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

Use these to derive alternative formulations for the following products,

$$\sin A \cos B, \quad \sin A \sin B, \quad \text{and} \quad \cos A \cos B$$

$\cos A \cos B$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

adding gives

$$2 \cos A \cos B = \cos(A-B) + \cos(A+B)$$

similarly

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

For $\sin A \cos B$,

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

summing

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

Product to Sum IDs

$$\cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$$

$$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$

$$\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$$

Evaluate Each Integral Using Any Applicable Method

(e) $\displaystyle\int \sin(2x)\cos(5x)\,dx$

$$= \int \frac{1}{2}\left(\sin(2x-5x) + \sin(2x+5x)\right) dx$$

$$= \frac{1}{2}\int \left(\sin(7x) - \sin(3x)\right) dx$$

$$= \frac{1}{2}\left(\frac{-1}{7}\right)\cos(7x) - \frac{1}{2}\left(\frac{-1}{3}\right)\cos(3x) + C$$

$$= \frac{1}{6}\cos(3x) - \frac{1}{14}\cos(7x) + C$$

Evaluate Each Integral Using Any Applicable Method

(f) $\displaystyle\int \frac{(2\cos t + 5)\sin t}{\cos^2 t - \cos t - 2}\, dt$

$u = \cos t$, $du = -\sin t\, dt$

$-du = \sin t\, dt$

$$= -\int \frac{2u+5}{u^2-u-2}\, du = -\int \frac{2u+5}{(u-2)(u+1)}\, du$$

$$\frac{2u+5}{(u-2)(u+1)} = \frac{A}{u-2} + \frac{B}{u+1}$$

$$2u+5 = A(u+1) + B(u-2)$$

If $u=2$, $4+5 = 3A \Rightarrow A = 3$

If $u=-1$, $-2+5 = -3B \Rightarrow B=-1$

$$-\int \frac{2u+5}{(u-2)(u+1)}\,du = -\int \left(\frac{3}{u-2} - \frac{1}{u+1}\right) du =$$

$$-3\ln|u-2| + \ln|u+1| + C$$

$$\int \frac{(2\cos t+5)\sin t}{\cos^2 t - \cos t - 2}\,dt = \ln|\cos t+1| - 3\ln|\cos t-2| + C$$

Evaluate Each Integral Using Any Applicable Method

(g) $\displaystyle\int \sin\sqrt{x}\,dx$

Let $y = \sqrt{x}$, $dy = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,dy = dx$

$2y\,dy = dx$

$= \int 2y \sin y \, dy$

$u = 2y$ $\quad du = 2\,dy$

$v = -\cos y$ $\quad dv = \sin y\,dy$

$= -2y\cos y + 2\int \cos y\,dy$

$= -2y\cos y + 2\sin y + C$

$$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$$

$$\int \frac{x}{x-1}\,dx$$

$u = x - 1 \qquad x = u+1$

$du = dx$

$$= \int \frac{u+1}{u}\,du = \int \left(1 + \frac{1}{u}\right) du$$

Evaluate Each Integral Using Any Applicable Method

(h) $\displaystyle\int \sec^{-1} x\, dx$

$u = \sec^{-1} x \qquad du = \dfrac{1}{x\sqrt{x^2-1}}\,dx$

$v = x \qquad dv = dx$

$$= x\sec^{-1} x - \int \frac{x}{x\sqrt{x^2-1}}\,dx$$

$$= x\sec^{-1} x - \int \frac{dx}{\sqrt{x^2-1}} = x\sec^{-1} x - \ln\left|x+\sqrt{x^2-1}\right| + C$$

(see work below)

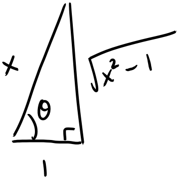
$$\int \frac{dx}{\sqrt{x^2-1}}$$

$$= \int \frac{\sec\theta \, \cancel{\tan\theta} \, d\theta}{\cancel{\tan\theta}}$$

$$= \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left| x + \sqrt{x^2-1} \right| + C$$



$x = \sec\theta$

$dx = \sec\theta \tan\theta \, d\theta$

$\sqrt{x^2-1} = \tan\theta$