February 7 MATH 1112 sec. 54 Spring 2020

Exponential and Logarithmic Functions

Recall, for $a > 0$ and $a \neq 1$

$$\log_a(x) = y \quad \Longleftrightarrow \quad a^y = x.$$

If $f(x) = a^x$, then

- The domain of f is $(-\infty, \infty)$.
- The range of f is $(0, \infty)$.

Question

If $g(x) = \ln(x)$, then which of the following is true?

(a) The domain of g is $(-\infty, \infty)$, and the range of g is $(-\infty, \infty)$.

(b) The domain of g is $(-\infty, \infty)$, and the range of g is $(0, \infty)$.

(c) The domain of g is $(0, \infty)$, and the range of g is $(0, \infty)$.

(d) The domain of g is $(0, \infty)$, and the range of g is $(-\infty, \infty)$.

Question

The value $\log_a(0)$ is

(a) equal to 1.

(b) only defined if $a > 1$.

(c) is always undefined.

(d) is only defined if $0 < a < 1$.

$\log_a(1) = 0$

Question

The graph of $y = \ln(x)$ is

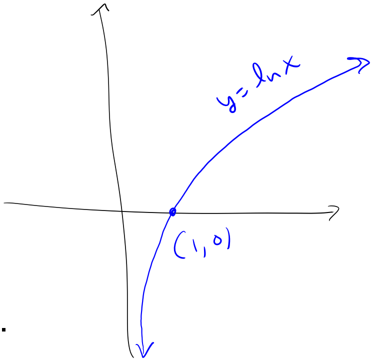
(a) increasing on $(0, \infty)$.

(b) has x-intercept at $(1, 0)$.

(c) has the y-axis as a vertical asymptote.

(d) All of the above are true about the graph.

(e) None of the above are true about the graph.



Example

Suppose $f(x) = \log_5(x+1) + \log_5(x-1)$ for all $x > 1$.

Find the inverse function $f^{-1}(x)$.

Let $y = f(x)$

$$y = \log_5(x+1) + \log_5(x-1)$$

Now, we isolate x.

Use $\log_5(M) + \log_5(N) = \log_5(MN)$ for any $M, N > 0$

$$y = \log_5\big((x+1)(x-1)\big)$$

$$(a+b)(a-b) = a^2 - b^2$$

$y = \log_5(x^2 - 1)$ $\qquad$ $y = \log_5 \star \Rightarrow \star = 5^y$

$$5^y = x^2 - 1$$

$$x^2 - 1 = 5^y$$

$$x^2 = 5^y + 1$$

$$x = \sqrt{5^y + 1}$$

Only the positive root makes sense since $x > 1$.

Swap labels $x \leftrightarrow y$

$$y = \sqrt{5^x + 1}$$

So $$f^{-1}(x) = \sqrt{5^x + 1}$$