Final Review MATH 2253 (Ritter)

(1) Evaluate the given limits.

(a) $\displaystyle \lim_{x\to 5} \frac{x^2 - 25}{x - 5}$

(b) $\displaystyle \lim_{x\to 0^-} \frac{|x|}{x}$

(c) $\displaystyle \lim_{t\to 9} \frac{x - 9}{3 - \sqrt{x}}$

(d) $\displaystyle \lim_{t\to 0} \frac{\csc 3t}{\csc 6t}$

(e) $\displaystyle \lim_{s\to \infty} \frac{\cos 2s}{s^2}$

(f) $\displaystyle \lim_{x\to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$

(g) $\displaystyle \lim_{x\to \pi} \frac{\cos x + 1}{x - \pi}$ HINT: Think about the definition of the derivative $\displaystyle f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

(h) $\displaystyle \lim_{h\to \frac{\pi}{4}} \frac{\tan h - 1}{h - \frac{\pi}{4}}$

(2) Compute the derivative of the given function.

(a) $f(t) = \cos t^2$

(b) $g(x) = \sqrt{x^2 - 4}$

(c) $f(x) = 7x \sin(3x+1)$

(d) $\displaystyle h(t) = \frac{\sqrt{t+1}}{t^2 - 1}$

(e) $\displaystyle F(x) = \int_{x^2}^{x^3} \cos t^2 \, dt$

(f) $h(x) = \sec(\tan x)$

(g) $\displaystyle G(s) = \int_0^{\cos s} \frac{dt}{\sin t}$

(h) $\displaystyle F(t) = \int_{\sqrt{t}}^{1} \frac{dx}{x}$

(3) Find $\frac{dy}{dx}$ using implicit differentiation.

(a) $\displaystyle xy^2 + \frac{x}{y} = 1$

(b) $\sin(x+y)-y^2 = 3x$

(c) $\tan x+\tan y = \sec y$

(d) $x^2y+y^2x = x+y$

(4) A ten foot ladder is leaning against a wall. The base starts to slide away from the wall at a constant 0.5 ft/sec. At what rate is the top sliding down the wall when the base is 6 ft from the wall? At what rate is the angle between the ladder and the ground changing at that same time?

(5) Suppose a box has a square base. If the sides of the base are increasing at a constant 1 in/sec, and the height is shrinking at a constant 2 in /sec, what is the rate at which the volume is changing when the base has side lenght 15 in, and the height is 20 in?

(6) Find the absolute max and min of the given function on the indicated interval.

$$g(x) = x^{3/5} \qquad -32 \leq x \leq 1$$

(7) Determine where the function is increasing, decreasing, concave up, and concave down. Identify all local maxima, minima, and points of inflection.

(a) $h(x) = 2x^3-18x$

(b) $g(x) = x^2\sqrt{5-x}$

(8) An enclosure is to be bounded on one side by a river. The other three sides are to be made from fencing. If 800m of fencing is available, what is the largest possible area that can be enclosed?

(9) What is the volume of the largest cylinder that can be inscribed in sphere of radius 20 units?

(10) Evaluate the given integrals.

(a) $\int_0^1 g(x)\,dx = 1, \quad \int_0^2 g(x)\,dx = 7, \quad \int_1^2 g(x)\,dx = ?$

(b) $\int_{-1}^2 x^2+3x-1\,dx$

(c) $\int_0^{\frac{\pi}{2}} \tan^3 \frac{x}{2} \sec^2 \frac{x}{2}\,dx$

(d) $\int_0^{\frac{\pi}{6}} \frac{\sin 2x}{\cos^4 2x}\,dx$

(e) $\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$

(f) $\displaystyle\int \frac{x^3}{\sqrt{x^4+1}}\,dx$

(g) $\displaystyle\int \cot^3 x \sec^2 x\,dx$

(h) $\displaystyle\int_0^{\frac{\pi}{4}} (1-\sin 2t)^{3/2} \cos 2t\,dt$

(11) Find the area between the curves $x = 2y^2$, $x = 0$, and $y = 3$.

(12) Find the volume of the solid generated by revolving the plane region in the previous problem about (a) the x-axis, (b) the y-axis.

(13) A 50lb block is hoisted to the top of a 100 ft building. If the cable used to lift the block weighs 2 lb/ft. Find the work done lifting the block.

(14) A spring obeying Hooke's law requires a force of 400 N to compress the spring from its natural length of 1 meter to a length of 75 cm. Determine the spring constant, and find the work done compressing the spring from its natural length of 1 meter to a length of 50 cm.

(15) Find any vertical and horizontal asymptotes to the graph of f given

(a) $f(x) = \dfrac{x^2+x}{x^3+x^2-2x}$

(b) $f(x) = \dfrac{2x^2-x^4}{3x^4+x^3}$

(c) $f(x) = \dfrac{2}{x+2} - \dfrac{3x}{x+4}$

(16) Explain what is (horribly!) wrong with the following expressions.

(a) $\dfrac{x}{x} = 1$

(b) $\dfrac{\sin x}{x} = \sin$

(c) $\dfrac{\sin x}{x} = 1$

(d) $\cos 3x = 3\cos x$

(e) $\cot = \dfrac{1}{\tan}$

(f) $\dfrac{1}{0} = 0$

(g) $\dfrac{3+x}{3} = 1+x$