Final Review MATH 2253 (Ritter) Solutions

(1) Evaluate the given limits.

(a) $\lim_{x\to 5} \frac{x^2-25}{x-5} = 10$

(b) $\lim_{x\to 0^-} \frac{|x|}{x} = -1$

(c) $\lim_{t\to 9} \frac{x-9}{3-\sqrt{x}} = -6$

(d) $\lim_{t\to 0} \frac{\csc 3t}{\csc 6t} = 2$

(e) $\lim_{s\to\infty} \frac{\cos 2s}{s^2} = 0$

(f) $\lim_{x\to 0} \frac{\sqrt{x^2+100}-10}{x^2} = \frac{1}{20}$

(g) $\lim_{x\to\pi} \frac{\cos x+1}{x-\pi} = 0$

(h) $\lim_{h\to\frac{\pi}{4}} \frac{\tan h-1}{h-\frac{\pi}{4}} = 2$

(2) Compute the derivative of the given function.

(a) $f'(t) = -2t\sin t^2$

(b) $g'(x) = \frac{x}{\sqrt{x^2-4}}$

(c) $f'(x) = 7\sin(3x+1)+21x\cos(3x+1)$

(d) $h'(t) = -\frac{3t^2+4t+1}{2\sqrt{t+1}(t^2-1)^2}$

(e) $F'(x) = 3x^2\cos(x^6)-2x\cos(x^4)$

(f) $h'(x) = \sec(\tan x)\tan(\tan x)\sec^2 x$

(g) $G'(s) = -\frac{\sin s}{\sin(\cos s)}$

(h) $F'(t) = -\frac{1}{2t}$

(3) Find $\frac{dy}{dx}$ using implicit differentiation.

(a) $\frac{dy}{dx} = \frac{y^4+y}{x-2xy^3}$

(b) $\dfrac{dy}{dx} = \dfrac{3 - \cos(x+y)}{\cos(x+y) - 2y}$

(c) $\dfrac{dy}{dx} = \dfrac{\sec^2 x}{\sec y \tan y - \sec^2 y}$

(d) $\dfrac{dy}{dx} = \dfrac{1 - 2xy - y^2}{x^2 + 2xy - 1}$

(4) The top is sliding down at $\frac{3}{8}$ ft/sec (has rate of change $-3/8$). The angle is decreasing at a rate of $\frac{1}{16}$ per second (it's derivative is $-1/16$).

(5) The volume is increasing at a rate of 150 in^3 at that instant.

(6) The absolute minimum is $g(-32) = -8$, and the absolute maximum is $g(1) = 1$.

(7) Determine where the function is increasing, decreasing, concave up, and concave down. Identify all local maxima, minima, and points of inflection.

(a) h is increasing on $(-\infty, \sqrt{3}) \cup (\sqrt{3}, \infty)$ and decreasing on $(-\sqrt{3}, \sqrt{3})$. It is concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$. h has an inflection at $(0, 0)$, a local maximum at $(-\sqrt{3}, 12\sqrt{3})$, and a local minimum at $(\sqrt{3}, -12\sqrt{3})$.

(b) The domain of g is $(-\infty, 5]$. g is increasing on $(0, 4)$ and decreasing on $(-\infty, 0) \cup (4, 5)$. It is concave up on $(-\infty, 4 - \sqrt{8/3})$ and concave down on $(4 - \sqrt{8/3}, 5)$. It has local maximum at $(4, 16)$ and local (actually global) minimum at $(0, 0)$. There is a point of inflection $(4 - \sqrt{8/3}, g(4 - \sqrt{8/3}))$. This is roughly $(2.37, 9.09)$.

(8) 80,000 square meters.

(9) $V_{max} = \dfrac{40^3\pi}{6\sqrt{3}}$

(10) Evaluate the given integrals.

(a) $\displaystyle\int_0^1 g(x)\,dx = 1, \quad \int_0^2 g(x)\,dx = 7, \quad \int_1^2 g(x)\,dx = 6$

(b) $\displaystyle\int_{-1}^2 x^2 + 3x - 1\,dx = \frac{9}{2}$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \tan^3\frac{x}{2}\sec^2\frac{x}{2}\,dx = \frac{1}{2}$

(d) $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\sin 2x}{\cos^4 2x}\,dx = \frac{7}{6}$

(e) $\displaystyle\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2} = \frac{1}{6}$

(f) $\displaystyle\int \frac{x^3}{\sqrt{x^4+1}}\,dx = \frac{1}{2}\sqrt{x^4+1}+C$

(g) $\displaystyle\int \cot^3 x \sec^2 x\,dx = -\frac{1}{2\tan^2 x}+C$

(h) $\displaystyle\int_0^{\frac{\pi}{4}} (1-\sin 2t)^{3/2}\cos 2t\,dt = \frac{1}{5}$

(11)18

(12) Find the volume of the solid generated by revolving the plane region in the previous problem about

(a) 81π, (b) $\dfrac{972\pi}{5}$

(13) 15,000 ft lbs

(14) $k = 1600$ N/m, and the work $W = 200$ J.

(15) Find any vertical and horizontal asymptotes to the graph of f given

(a) Vert. $x=-2$, $x=1$, Hor. $y=0$

(b) Vert. $x=0$, $x=-\dfrac{1}{3}$, Hor. $y=-\dfrac{1}{3}$

(c) Vert. $x=-2$, $x=-4$, Hor. $y=-3$

(16) Explain what is (horribly!) wrong with the following expressions.
(a) What if $x=0$?
(b) This is crazy! The sine is a function; we can't just divorce it from its argument.
(c) This is always false. There is no x-value for which this is true.
(d) Coefficients of arguments of trigonometric functions cannot be *factored* out. This requires a property that the cosine doesn't have.
(e) The tangent and cotangent are functions requiring arguments. A cot is something you lie on, and a tan is what you get out in the sun.
(f) This is also crazy. The left side isn't defined.
(g) The conditional statement has a single solution, but this is not an identity. It appears to rely on illegitimate algebra.