Extra Review Questions for the Final Exam: Math 2335 (Ritter)

The final will be comprehensive. These questions cover section 5.4 and chapter 6 material not covered on previous exams.

(1) Use the central difference formula to approximate $f'(0)$ for $h = 0.1, 0.05$, and 0.025 where

(a) $f(x) = \sin(x)$

(b) $f(x) = \tan^{-1}(2x)$

(c) $f(x) = xe^{-x}$

(2) Use the method of undertermined coefficients to find an approximation to the second derivative of the form

$$f''(x) \approx Af(x + 3h) + Bf(x + 2h) + Cf(x + h) + Df(x).$$

Determine the order of the method.

(3) A function f is known to satisfy the two conditions

$$f'(x) = x \ln(f(x)), \quad f(1) = e$$

Use the forward difference approximation to $f'(x)$ to approximate $f(1.1)$, $f(1.2)$ and $f(1.3)$.

(4) Verify that the given matrices L and U constitute an LU decomposition of the matrix A.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

(5) Consider the following linear system $Ax = b$.

$$\begin{aligned} 2x_1 &- x_2 &+ x_3 &= -5 \\ x_1 & &+ 2x_3 &= -5 \\ -x_1 &+ x_2 &- x_3 &= 4 \end{aligned}$$

(a) Construct the coefficient matrix A.

(b) Find an LU decomposition $A = LU$.

(c) Solve $L\mathbf{g} = \mathbf{b}$.

(d) Find (x_1, x_2, x_3) by solving $U\mathbf{x} = \mathbf{g}$.

(6) Show that the matrix A does not have an LU decomposition.

$$
A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 1 & -2 \\ 6 & 1 & 3 \end{bmatrix}.
$$

(7) Show that the matrix PA does have an LU decomposition where A is the matrix from problem (5) and P is given below. Find the LU decomposition.

$$
P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}
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