Extra Review Questions for the Final Exam: Math 2335 (Ritter)

The final will be comprehensive. These questions cover section 5.4 and chapter 6 material not covered on previous exams.

(1) Use the central difference formula to approximate $f'(0)$ for $h = 0.1, 0.05$, and 0.025 where

(a) $f(x) = \sin(x)$ $\quad D_{0.1}f(0) = 0.99833, \quad D_{0.05}f(0) = 0.99958, \quad D_{0.025}f(0) = 0.99990$

(b) $f(x) = \tan^{-1}(2x)$ $\quad D_{0.1}f(0) = 1.97396, \quad D_{0.05}f(0) = 1.99337, \quad D_{0.025}f(0) = 1.99834$

(c) $f(x) = xe^{-x}$ $\quad D_{0.1}f(0) = 0.90484, \quad D_{0.05}f(0) = 0.95123, \quad D_{0.025}f(0) = 0.97531$

(2) Use the method of undertermined coefficients to find an approximation to the second derivative of the form

$$f''(x) \approx Af(x+3h) + Bf(x+2h) + Cf(x+h) + Df(x).$$

Determine the order of the method.

$$D_h^{(2)}f(x) = \frac{-f(x+3h) + 4f(x+2h) - 5f(x+h) + 2f(x)}{h^2}, \quad \text{It's an order 2 method.}$$

(3) A function f is known to satisfy the two conditions

$$f'(x) = x\ln(f(x)), \quad f(1) = e$$

Use the forward difference approximation to $f'(x)$ to approximate $f(1.1)$, $f(1.2)$ and $f(1.3)$.

$f(1.1) \approx 2.81828$, $f(1.2) \approx 2.93226$, $f(1.3) \approx 3.06135$

(4) Verify that the given matrices L and U constitute an LU decomposition of the matrix A.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

(5) Consider the following linear system $A\mathbf{x} = \mathbf{b}$.

$$\begin{array}{rcrcrcr} 2x_1 & - & x_2 & + & x_3 & = & -5 \\ x_1 & & & + & 2x_3 & = & -5 \\ -x_1 & + & x_2 & - & x_3 & = & 4 \end{array}$$

(a) Construct the coefficient matrix A. $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 2 \\ -1 & 1 & -1 \end{bmatrix}$

(b) Find an LU decomposition $A = LU$. $L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 & -1 & 1 \\ 0 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & -2 \end{bmatrix}$

(c) Solve $L\mathbf{g} = \mathbf{b}$. $\mathbf{g} = \left(-5, -\frac{5}{2}, 4\right)$

(d) Find (x_1, x_2, x_3) by solving $U\mathbf{x} = \mathbf{g}$. $\mathbf{x} = (-1, 1, -2)$

(6) Show that the matrix A does not have an LU decomposition.

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 1 & -2 \\ 6 & 1 & 3 \end{bmatrix}.$$

(7) Show that the matrix PA does have an LU decomposition where A is the matrix from problem (5) and P is given below. To show this, find the LU decomposition.

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 3 & 1 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & -4 \end{bmatrix}$$