Critical Evaluation of Mathematics

Many of the *Common Core Standards* for mathematics include a standard of the form

Construct viable arguments and critique the reasoning of others.

This project involves looking at some mathematical arguments that lead to some pretty weird (and downright false!) conclusions. The challenge is to figure out where the author went wrong. The *bad math* can be difficult to spot, so it takes a very careful eye.

Example 1: A "proof" that $1 = 2$

Let $a = b$. Then

$$a^2 = ab \tag{1}$$

$$a^2 - b^2 = ab - b^2 \tag{2}$$

$$(a+b)(a-b) = b(a-b) \tag{3}$$

$$a + b = b \tag{4}$$

$$b + b = b \tag{5}$$

$$2b = b \tag{6}$$

$$2 = 1 \tag{7}$$

Okay, so we know that's not right! Can you identify which step(s) is(are) invalid? Are there any important mathematical *facts* illustrated by this little exercise?

Example 2: A ”proof” that $\pi = 3$

Let $x = (\pi + 3)/2$. Then

$$\begin{aligned}
2x &= \pi + 3 && (8)\\
2x(\pi - 3) &= (\pi + 3)(\pi - 3) && (9)\\
2\pi x - 6x &= \pi^2 - 9 && (10)\\
9 + 2\pi x - 6x &= \pi^2 && (11)\\
9 - 6x &= \pi^2 - 2\pi x && (12)\\
9 - 6x + x^2 &= \pi^2 - 2\pi x + x^2 && (13)\\
(3 - x)^2 &= (\pi - x)^2 && (14)\\
3 - x &= \pi - x && (15)\\
3 &= \pi && (16)
\end{aligned}$$

Again, we know this is just not true! Yet almost all of the steps above are valid—*almost* all! Where is the bad math? Can you find an important principle of algebra illustrated here?

Example 3: A "proof" that $-1 = 1$

(assumes knowledge of the imaginary unit i with the property $i^2 = -1$)

We know that $-1 = -1/1 = 1/-1$, so

$$\frac{-1}{1} = \frac{1}{-1} \tag{17}$$

$$\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}} \tag{18}$$

$$\frac{\sqrt{-1}}{\sqrt{1}} = \frac{\sqrt{1}}{\sqrt{-1}} \tag{19}$$

$$\frac{i}{1} = \frac{1}{i} \tag{20}$$

$$i^2 = 1^2 \tag{21}$$

$$-1 = 1 \tag{22}$$

Nowhere on Earth does this make sense! Can you find the flaw in this argument? (The bad math in this example is similar to that of example 2.) Can you identify an important mathematical symbol being used carelessly?

Remember that all of these arguments are based on bad, illigitimate, untrue, fake, counterfeit ...(you get the picture) mathematical manipulations. They make great party gags, but please don't mistake them for important information about the world—other than the fact that bad math can lead to crazy conclusions!