Glossary[1]

Line: A line is the collection of points (x, y) in the plane that satisfy an equation of the form

$$Ax + By = C$$

for fixed real numbers A, B, and C with at least one of A or B nonzero.

Slope: The *slope* of a line is a measure of the *rate of change* in the y-value as it depends on a change in the x-value between two points on a line. (The slope of any non-vertical line is constant.)

Parallel: Two lines are *parallel* if the have the same slope (or if both are vertical).

Perpendicular: Two lines with defined slopes m_1 and m_2 are *perpendicular* if $m_1 m_2 = -1$. In addition, a vertical line is perpendicular to any line of slope zero.

Relation: A *relation* is a correspondence (or a mapping) between a first set, called the *domain*, and a second set, called the *range*, such that each element in the domain corresponds to (is related to/is mapped to) at least one element in the range.

Function: A *function* on two sets is a relation in which each element of the domain corresponds to (is related to/is mapped to) exactly one element in the range.

Algebra of Functions: Let f and g be functions, and suppose that x is in the domain of each. Then define $f + g$, $f - g$, fg and f/g, and use the following notation

- $(f + g)(x) = f(x) + g(x)$
- $(f - g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x)g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ provided $g(x) \neq 0$

Composition of Functions: Let f and g be functions. Then the *composite* function denoted

$$f \circ g,$$

also called the *composition* of f and g, is defined by

$$(f \circ g)(x) = f(g(x)).$$

Even Function: A function f is called an *even function* if

$$f(-x) = f(x)$$

for each x in its domain. We can say that such a function has *even symmetry*.

Odd Function: A function f is called an *odd function* if

$$f(-x) = -f(x)$$

for each x in its domain. We can say that such a function has *odd symmetry*.

[1]The entries are ordered according to course content as opposed to alphabetically.

Polynomial: A *polynomial* in a variable x is an expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,$$

where n is a nonnegative integer, and the numbers $a_0, a_1, \ldots, a_n$ are called the *coefficients*.

Polynomial Equation/Polynomial Function: A *polynomial equation* is an equation that can be written in the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 = 0.$$

A *polynomial function* is one that can be written in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0.$$

Quadratic Function: A quadratic function is a second degree polynomial $f(x) = ax^2 + bx + c$ where $a \neq 0$. f is said to be in *vertex form* when expressed as $f(x) = a(x-h)^2 + k$ where (h, k) is the vertex of the graph (a parabola) of f.

Quadratic Equation/ Root/ Zero: A quadratic equation in x is one that can be written in the form

$$ax^2 + bx + c = 0, \quad \text{where} \quad a \neq 0. \tag{1}$$

We will say that the above is written in **standard form**.

If (1) has a real number solution x_0, then this number is called

- a *root* of the equation (1).
- It is also called a *zero* of the associated quadratic function $f(x) = ax^2 + bx + c$.
- A zero of $f(x) = ax^2 + bx + c$ is the location of an x-**intercept** to the graph of f.

Discriminant (of a quadratic): The discriminant of the quadratic $ax^2 + bx + c$ is $b^2 - 4ac$.

Irreducible (Quadratic): A quadratic polynomial is called *irreducible* if it's discriminant is negative. An irreducible quadratic cannot be realized as the product of linear factors having only real coefficients.

Zero Multiplicity: Suppose that $(x-c)^k$ is a factor of a polynomial $P(x)$ and $(x-c)^{k+1}$ is not a factor of P. Then c is called a *zero of multiplicity* k of the function P. We may also call it a *root of multiplicity* k of the polynomial.

Turning Point: A *turning point* on the graph of a function f is a point at which f changes from increasing to decreasing or from decreasing to increasing. Note that a turning point is a relative maximum or a relative minimum.

Rational Funcion: The function $f(x)$ is a *rational function* if f has the form

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are both polynomials (and $q(x)$ is not the zero function).

Vertical Asymptote: The vertical line $x = a$ is a vertical asymptote of the function f if $f(x)$ increases without bound as x approaches the number a.

Lowest Terms (rational function): The rational function

$$f(x) = \frac{p(x)}{q(x)}$$

is in *lowest terms* if the polynomials p and q have no common factors.

Horizontal Asymptote: The horizontal line $y = b$ is a horizontal asymptote to the function f if the graph of $f(x)$ hugs the line $y = b$ as x becomes unbounded. Symbolically

$$f(x) \to b \quad \text{as} \quad x \to \infty \quad \text{or as} \quad x \to -\infty.$$

Inverse Relation: Let S be a relation with domain D and range R. The inverse relation S^{-1} is the relation having domain R and range D defined by[2]

$$(x, y) \in S^{-1} \quad \text{provided} \quad (y, x) \in S.$$

One to One: A function f is *one to one* if different inputs have different outputs. That is f is one to one provided

$$a \neq b \quad \text{implies} \quad f(a) \neq f(b).$$

Equivalently, f is a one to one function provided

$$f(a) = f(b) \quad \text{implies} \quad a = b.$$

Exponential Function: Let a be a positive real number different from 1—i.e. $a > 0$ and $a \neq 1$. The function

$$f(x) = a^x$$

is called the *exponential function of base* a. Its domain is $(-\infty, \infty)$, and its range is $(0, \infty)$.

Logarithm Base a: Let $a > 0$ and $a \neq 1$. For $x > 0$ define $\log_a(x)$ as a number such that

$$\text{if} \quad y = \log_a(x) \quad \text{then} \quad x = a^y.$$

The function

$$F(x) = \log_a(x)$$

is called the *logarithm function of base* a. It has domain $(0, \infty)$, range $(-\infty, \infty)$, and if $f(x) = a^x$ then

$$F(x) = f^{-1}(x).$$

Exponential Growth/Decay: When a quantity Q changes in time according to the model $Q(t) = Q_0 e^{kt}$, it is said to experience *exponential growth* ($k > 0$) or *exponential decay* ($k < 0$). Here, Q_0 is the constant initial quantity $Q(0)$, and k is a constant.

Trigonometric Ratios of an Acute Angle: Let θ be an acute angle in a right triangles whose adjacent leg length is called "adj", opposite leg length is called "opp" and whose hypotenuse length is called "hyp". The trigonometric values of θ are given by

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$, read as "sine theta"

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$, read as "cosine theta"

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$, read as "tangent theta"

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{1}{\sin\theta}$, read as "cosecant theta"

[2]Recall that $\in$ means "in," so $(x, y) \in S^{-1}$ is read "(x, y) is an element of S^{-1}."

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos\theta}, \qquad \text{read as "secant theta"}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\tan\theta}, \qquad \text{read as "cotangent theta"}$$

Complementary Angles Two acute angles whose measures sum to $90°$ are called *complementary angles*. Given an acute angle θ its *complement* is the angle $90° - \theta$.

Angle, Standard Position, Initial and Terminal Ray: We define an angle by a pair of rays (say R_1 and R_2) that share a common origin. We can indicate direction for an angle by indicating one ray as the *initial ray* (starting) and the other as the *terminal ray* (ending). An angle is said to be *positive* when the rotation from intial to terminal ray is counter clockwise, and *negative* when this rotation is clockwise.
An angle is in **Standard position** in a Cartesian coordinate plane if its vertex is at the origin, and its initial ray is along the positive x-axis.

Coterminal Angles: Have the same initial and terminal rays when in standard position.

Degree Measure: Degrees are a measure of angles for which one circle is $360°$.

Supplementary Angles: Two positive angles whose measures sum to $180°$ are called *supplementary* angles.

Quadrantal Angle: A quadrantal angle is one whose terminal side is concurrent with one of the coordinate axes when in standard position.

Reference Angle: Let θ be an angle in standard position that is not a quadrantal angle. The *reference angle* θ' associated with θ is the angle of measure $0° < \theta' < 90°$ between the terminal side of θ and the *nearest* part of the x-axis.

Trigonometric Functions of any Angle: Let θ be an angle in standard position, and let (x, y) be a point on its terminal ray (other than the origin). Set $r = \sqrt{x^2 + y^2}$, the distance between the point (x, y) and the origin. Then

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y} \quad \text{provided } y \neq 0$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x} \quad \text{provided } x \neq 0$$

$$\tan\theta = \frac{y}{x} \quad \text{provided } x \neq 0 \qquad \cot\theta = \frac{r}{y} \quad \text{provided } y \neq 0$$

Central Angle: An angle is called a central angle in relation to a circle when the angle's vertex is at the center of the circle.

Radians: (Rad) An angle is measured in radians in relation to a unit circle (circle of radius 1). A central angle $\theta = 1$ radian if the angle subtends an arc in a unit circle of length 1. The measure of a full circle angle is 2π.

Some Special Angle Terms:

- An **acute** angle is between $0°$ and $90°$ (0 and $\frac{\pi}{2}$).
- An **obtuse** angle is between $90°$ and $180°$ ($\frac{\pi}{2}$ and π).
- A **right** angle has measure $90°$ ($\frac{\pi}{2}$).
- A **reflex** angle has measure between $180°$ (π) and $360°$ (2π).
- **Quadrantal** angles are integer multiples of $90°$ ($\frac{\pi}{2}$)
- A **straight** angle has measure $180°$ (π).

Arclength & Sector Area: Given a circle of radius r, the length s of the arc subtended by the (positive) central angle θ (in radians) is given by

$$s = r\theta.$$

The area of the resulting sector is $A_{sector} = \frac{1}{2}r^2\theta$.

Angular Speed If an object moves along the arc of a circle through a central angle θ in the time t, the angular speed is denoted by ω (lower case omega) and is defined by

$$\omega = \frac{\theta}{t} = \frac{\text{angle moved through}}{\text{time}}.$$

Linear Speed If the circle has radius r, then the distance traveled is the arclength $s = r\theta$. The linear speed is denoted by ν (lower case nu) and is defined by

$$\nu = \frac{s}{t} = \frac{r\theta}{t} = r\omega.$$

Trigonometric Function of a Real Variable: Let s be any real number and consider the central angle in standard position, of measure s, in the unit circle. The coordinates of the intersection of the terminal ray and the unit circle are $(x, y) = (\cos s, \sin s)$. Moreover, $\tan s = \frac{\sin s}{\cos s}$ provided $\cos s \neq 0$. The cosecant, secant, and cotangent are defined as the reciprocals of the sine, cosine and tangent, respectively.

Periodic & Fundamental Period: A function f is said to be *periodic* if there exists a positive constant p such that

$$f(x + p) = f(x)$$

for every x in the domain of f. The smallest such number p is called the *fundamental period* of the function f.

Amplitude: The sine and cosine functions oscillate between their maximum and minimum values. Half of the distance between the maximum and minimum is called the *amplitude*. This is also the distance between the x intercepts and the extreme values for $y = \sin x$ and $y = \cos x$.

Frequency: The reciprocal of the period is called the *frequency*.

Phase Shift: A horizontal shift for a trigonometric function is called a *phase shift*.

Inverse Sine Function: For x in the interval $[-1, 1]$ the inverse sine of x is denoted by either

$$\sin^{-1}(x) \quad \text{or} \quad \arcsin(x)$$

and is defined by the relationship

$$y = \sin^{-1}(x) \quad \Longleftrightarrow \quad x = \sin(y) \quad \text{where} \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}.$$

Inverse Cosine Function: For x in the interval $[-1, 1]$ the inverse cosine of x is denoted by either

$$\cos^{-1}(x) \quad \text{or} \quad \arccos(x)$$

and is defined by the relationship

$$y = \cos^{-1}(x) \quad \Longleftrightarrow \quad x = \cos(y) \quad \text{where} \quad 0 \leq y \leq \pi.$$

Inverse Tangent Function: For all real numbers x, the inverse tangent of x is denoted by

$$\tan^{-1}(x) \quad \text{or by} \quad \arctan(x)$$

and is defined by the relationship

$$y = \tan^{-1}(x) \quad \iff \quad x = \tan(y) \quad \text{where} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}.$$

Expression (Mathematical): An expression is a grouping of numbers, symbols, and/or operators that may define a mathematical object or quantity. An expression is a *NOUN*.

Statement (Mathematical): A mathematical statement is an assertion that may be true or false. A statement is a *SENTENCE*.

Conditional Statement (Mathematical): A conditional statement is a statement that may be true under certain conditions but is false when such conditions are not met. Typically, it is true when a variable is assigned a certain value/values. But it is false when other values are assigned.

Identity (Mathematical): An identity is a mathematical statement that is ALWAYS true. If an identity is stated as an equation, this means that for every value of any variable such that both sides of the equation are defined, both sides of the equation are the same.