Sample Entries for a Linear Algebra Glossary

1. Linear Equation: Suppose we have n variables $x_1, x_2, \ldots, x_n$. A Linear Equation in these variables is an equation of the form

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

where $a_1, \ldots, a_n$ are called the coefficients. Typically, the coefficients and the right side b are known numbers. The main characteristics of the equation are that the variables appear only to the first power (no squares, square roots, etc.), they are multiplied by coefficients (no multiplying variables), and added. Examples could include

$$2x_1 + 4x_2 - x_3 = 7 \quad \text{or} \quad 5x - 2y + z - w = -16.$$

The variables can have different characters or use subscripts. An example of an equation that is NOT linear is $x^2 + y^2 = 1$ because of the squares, or $xyz = 3$ because the variables are multiplied.

18. Span: The Span of a set of vectors is the set of all linear combinations of those vectors. If $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ is a set of vectors, the span of this set would be denoted by

$$\text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\},$$

and would be the set of all vectors of the form $c_1 \mathbf{u}_1 + c_2 \mathbf{u}_2 + \ldots + c_k \mathbf{u}_k$ where the coefficients $c_1, \ldots, c_k$ are any scalars. For example, if we have two vectors, $\mathbf{u}$ and $\mathbf{v}$, then $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is a set of infinitely many vectors that can be written as

$$c_1 \mathbf{u} + c_2 \mathbf{v}$$

where c_1 and c_2 can be any real numbers. And, if we say that the vector $\mathbf{w}$ is in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$, then we know that there must be a pair of numbers c_1 and c_2 such that $\mathbf{w} = c_1 \mathbf{u} + c_2 \mathbf{v}$.