Aug. 28 Math 2253H sec. 05H Fall 2014

Intermediate Value Theorem (IVT) Suppose f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$. Then there exists c in the interval (a, b) such that $f(c) = N$.

Conceptually: If f is continuous, and it takes on two different values $f(a)$ and $f(b)$, then it must take on all of the values between them.

IVT

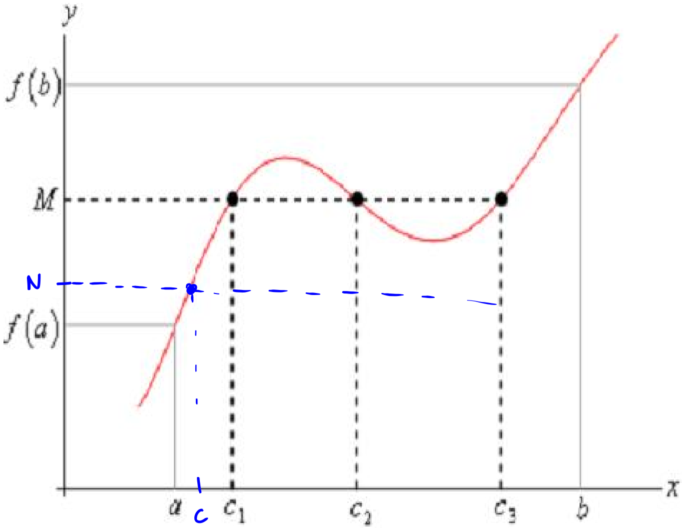

Figure: Graphical depiction of the Intermediate Value Theorem

IVT: Challenge

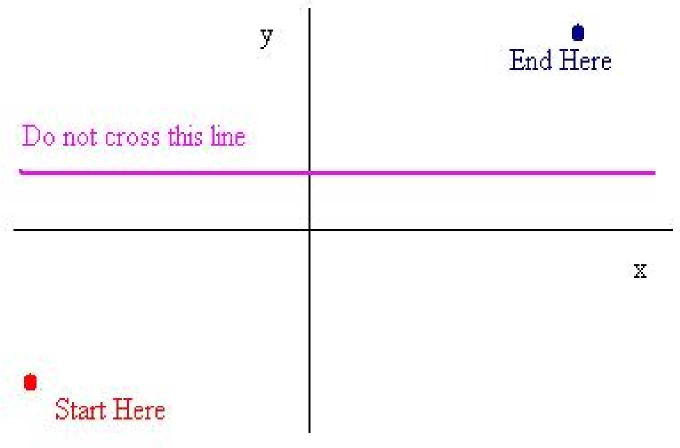

Figure: Draw the graph of a **continuous function**. Start at the red dot, end at the black dot. **BUT DON'T CROSS THE PINK LINE!**

Application of the IVT

Show that there exists a number that is exactly one less that it's fifth power.

Let the number be x. This number must satisfy $x = x^5 - 1$.

Note: $x = x^5 - 1$ is equivalent to $x - x^5 + 1 = 0$

Let $f(x) = x - x^5 + 1$

A solution to our equation would be an x-intercept of f.

f is continuous everywhere (its a polynomial!)

$f(x) = x - x^5 + 1$

Note $f(0) = 0 - 0 + 1 = 1$

$f(5) = 5 - 5^5 + 1 = 6 - 5^5$

$f(0) > 0$ and $f(5) < 0$

So zero is between $f(0)$ and $f(5)$

f is continuous on $[0, 5]$.

By the IVT, there must exist

Some c in $(0,5)$ such that

$$f(c) = 0.$$

$$0 = f(c) = c - c^5 + 1 \implies c = c^5 - 1$$

c is a number that is one less than its 5^{th} power.

Application of the IVT

Use the IVT to show that there is a solution of the equation on the specified interval.

$$\sin x = x^2 - x, \quad \text{on} \quad [1, 2]$$

Let $f(x) = \sin x - x^2 + x$. Then a solution to the equation would correspond to an x-intercept of f.

Note f is continuous everywhere – so it's continuous $[1, 2]$.

$f(1) = \sin(1) - 1^2 + 1 = \sin(1) \qquad f(1) > 0$

$f(2) = \sin(2) - 2^2 + 2 = \sin(2) - 2 \qquad f(2) < 0$

Zero is a number between $f(1)$ and $f(2)$.

By the IVT, there exists c in $(1, 2)$

such that $f(c) = 0$

For this number c

$$0 = f(c) = \sin c - c^2 + c \Rightarrow$$

$$\sin c = c^2 - c$$